Selecting Value-at-Risk Models for Government of India Fixed Income Securities[&]

by

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Abstract: This paper has evaluated a number of available VaR models, such as, variance-covariance/normal (including Risk-Metric approach), historical simulation and tail-index (Hill's estimator) based method for estimating VaR for a number of selected GOI bonds and representative portfolios of GOI bonds for banks and PDs. Competing VaR methods/strategies are evaluated through backtesting and assessment of two typical loss-functions. Empirical results we present are quite interesting. It is seen that normal methods (including Risk-Metric approach) generally under-estimate VaRs. On the other hand, VaR models based on HS and tail-index (using Hill's estimator) are quite good, though the later produces slightly more conservative VaR estimates. But when we look at the loss-functions, tail-index method appears to give the least magnitude/amount of excess loss (i.e. loss over estimated VaR). These results, however, are tentative. One needs to experiment with alternative sizes of rolling sample to check the robustness of the results. Future research may also investigate on appropriate formulation of loss-function while evaluating VaR models.

Key Words: Market Risk, Value-at-Risk, VaR Models, Back Testing, Evaluation of VaR Models.

[&] *Views expressed in this paper are purely personal and not necessarily be of the organisations to which the authours belong. This is a preliminary version.*

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1. Introduction

In recent years, the concept of Value at Risk (VaR) occupies the central role in managing market risk. Since the Basle Committee Report (1995, 1996) of the Banks for International Settlements (BIS), many central banks have made it mandatory to their supervised banks for quantifying market risk through VaR and to maintain minimum required capital for this quantified risk. However, no specific VaR model is recommended by regulators (BIS or central banks). Supervised banks (henceforth simply banks) are free to use their own VaR models and the approach so prescribed by the regulators is called as 'internal model approach'.

From a bank's point of view, choosing appropriate VaR model assumes importance as the required capital charge for market risk is linked to VaR estimates. Larger is the value of VaR estimate, higher is the amount of required capital. But banks generally do not want to have higher capital charge. It is argued (see for instance, Wong, et al. 2003) that an increase in required capital increases the equity-asset ratio of bank. As equity tends to have higher required rate of returns than debt or other sources of capital, the average cost of capital for the bank will rise. This may result in low profitability of the bank in terms of return on equity. Thus, banks may have a tendency/preference towards a model that produces lower VaR. In this process, however, banks may be exposed to risk beyond their capacity and may be vulnerable to the shocks arising out market swings (i.e. due to market risk). In order to eliminate such (excessive) risk-taking activities of banks, regulators provide certain norms (such as backtesting, data period and other factors for VaR estimations, etc.) to be satisfied by the VaR estimates.

The selection of an appropriate VaR model in reality, however, is a difficult task and each bank faces a problem of choosing one amongst several available alternatives. A simple guide to banks in this purpose would be to choose a model that produces as minimum VaR as possible and also satisfy the regulatory requirements/norms prescribed by the regulators/Basle Committee. One important point here is that these regulatory requirements of VaR estimates actually focuses on the frequency of the events of loses above estimated VaR but does not consider the magnitudes of failures/excess-loses. In a real world, however, a bank may have its own objective function that need not always be same as (or consistent with) the regulatory norms. For example, a bank manager may prefer to accept a model that results in small additional loses (i.e. the loss in excess of estimated VaR) even if it shows poor performance in backtesting than a model, which passes through the backtesting but results in very high additional loses. This may particularly be so if it happens that the total additional loss in the former case is less than that in the later case. In other words, banks may prefer even a model that fails in backtesting but results in relatively less loss (after making adjustment for the higher multiplying factor for failing in back testing). Keeping these points in mind, some researchers have proposed to minimise certain loss functions while making a choice of a VaR model from various alternatives (Lopez, 1999; Sarma, et al., 2003).

In India, about 58-68 percent (in percent of outstanding rupees) of GOI bonds are held by commercial banks in recent years (Nath and Samanta, 2003). Therefore, it is important for banks to select VaR models for managing the market risks in respect to GOI bonds in their portfolios. In this connection, Darba (2001) attempts to estimate VaRs for select portfolios of fixed income instruments held by ten Primary Dealers (PDs)[1]. He compares three methods of VaR estimation, namely, Normal method, Historical Simulation (HS) and Extreme Value Theory (EVT) and concludes that EVT produces best VaR estimator in terms of correct failure ratio and lowest VaR for the representative portfolios considered. However, he himself points out that the results need to be checked over all PDs. In a recent study, Nath and Samanta (2003) have experimented with several alternative VaR models, such as, normal method (including Risk Metric), HS and Tail-Index based approach for two hypothetical portfolios of GOI bonds as well as selected 31 individual GOI bonds. They found that normal method generally underestimate VaR, leading to more frequent VaR violations in backtesting than other competing models. In both these studies, VaR models have been validated only through 'backtesting' criterion. As stated earlier, backtesting ignores magnitude of VaR violations. So, in real world, portfolio managers may have their own objective function that is different from that implied by the 'backtesting' (Lopez, 1998, 1999; Sarma, et al., 2003). In the present paper, we have demonstrated a framework for selecting appropriate VaR model for two representative portfolios of Government of India (GOI) fixed income securities. We also examine various VaR models for 31 individual GOI bonds. This study is different from Darba (2001) and Nath and Samanta (2003) in that it focuses on a wider class of evaluation, criteria, viz., (i) backtesting, (ii) statistical tests of VaR accuracy (considering separately, unconditional coverage and conditional coverage), and also (iii) assessment using typical loss-functions, which a portfolio manager would like to minimise.

The organisation of rest of the paper is as follows. In Section 2, we present a few available VaR estimation techniques. Section 3 discusses various criteria/objective functions for selecting/evaluating VaR model. The database on GOI bonds used in

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[1] Supplied by Primary Dealers Association. For the sake of confidentiality, however, Darba (2001) did not disclose any information on these ten portfolios.

this empirical study is discussed in Section 4. Section 5 presents the empirical results. Finally, Section 6 concludes the paper.

2. Select VaR Models

The VaR represents the maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence in probabilistic term. Holding period is one of the most key elements in VaR estimation and the same is chosen on the basis of time that an organization would take to liquidate its position if the need arises. In a very liquid market, 1-day holding period may seem to be justified while in an illiquid market; it may take even more than 10 days to liquidate the portfolio. The amount of capital charge for market risk is linked to VaR estimates and hence will be different for different holding periods.

2.1. Direct Estimation of VaR

The VaR for k-days holding period can be computed directly from k-days return series. The VaR for multi-days holding period can also be calculated/approximated indirectly from the estimated VaR for one-day holding period[2]. We consider here the case of direct estimation of VaR for one-day holding period from daily return series.

From statistical point of view, the problem of VaR estimation ultimately boils down to the estimation of quantile of return distribution. If the underlying distribution were normal one would simply estimate the required quantile with the help of quantiles of standard normal distribution (which are known to us) and estimated mean and variance of the return. Of course, one has to decide upon the type of distribution (i.e. whether conditional or unconditional/static) and the type of sample (i.e. whether full sample or rolling sample). But the biggest problem in estimating VaR has been the non-normality of return distribution due to leptokurtic and/or skewness problems. Though the normality assumption of conditional heteroscedasticity models, primarily used for modelling the phenomenon of volatility clustering, can partly handle with the leptokurtic behaviour of unconditional distribution of return, utility of such models in estimating VaR is seen not very impressive (Wong et al., 2003). Two alternative strategies exist to handle with non-normal distribution. First, to adopt some non-parametric methods, say Historical Simulation (HS), to estimate the quantiles of return distribution. Second, one can fit appropriate form of parametric distribution, either fitting suitable non-normal distribution (say, by fitting t-distribution, suitable mixture distribution or so) for entire portion of the distribution or by modelling only the extreme-observations of the distribution under extreme value theory (which also includes tail-index based analysis).

As in Nath and Samanta (2003) we consider eight competing VaR models; five under normal method (viz., full sample estimates, rolling sample estimates and Risk-Metric approach for three alternative decay factors), two under Historical Simulation, and two under tail-index based approach using Hill's estimator (viz., full sample and rolling sample estimates). A discussion on these models is presented below.

[2] In practice, generally VaR with one-day holding period is calculated daily basis from daily return series and VaRs for multi-days holding period are calculated indirectly using the estimates of one-day VaRs.

2.1.1. Variance-Covariance (Normal) Method

The Variance-Covariance (Normal) method is the easiest of the VaR methodologies, which assumes normality of return distribution, either conditional or unconditional. As normal distribution is characterized by first two moments (say, mean and variance), the main task involved in estimating VaR under normal method has been to estimate mean and standard deviation of return distribution. But there may be several alternatives strategy in terms of modelling/estimating variance. Particularly, whether to take static variance of the entire time series or conditional variance is a point for debate. Moreover, even if the debate is resolved, there would be several strategy for estimating required parameters.

When we assume normality of unconditional distribution, mean/variance can be estimated either by using full sample data or a rolling sample data. To illustrate the issue, suppose we are estimating VaR for time point t. Now for the 'full sample' case, all estimates are obtained by using all data from time points 1 to t. In the case of rolling sample, however, one makes use of data for the time points (t-k+1) to t, where k is a positive integer representing the size of the rolling sample. In either case, if μ and σ^2 denote the mean and variance of the return distribution, then the required quantile for probability p, denoted by ξ_p is given by

$$\xi_p = \mu + \tau_p \sigma \qquad \text{..... (1)}$$

where, τ_p denotes the p-th quantile of standard normal distribution.
The variance can also be modelled conditionally (what is done for handling the volatility clustering phenomenon observed in financial markets returns). In this case, it is assumed that the distribution of return series given all past information follows a normal distribution, with the variance being a function of past observations/residuals. Thus, variance here is conditionally heteroskedastic. The popular Risk Metric approach (J.P.Morgan, 1996) basically estimate VaR by modelling volatility clustering phenomenon in a simplified/restricted form and more general form of conditional heteroskedasticity can be modelled through the class of ARCH/GARCH models (Engle, 1982; Boller, 1986). Interestingly, however, Wong et al. (2003) report that though conditional heteroscedastic models are very useful for better capturing volatility clustering phenomenon, they do not necessarily produce good VaR estimates. For the sake of completeness of the discussion, we, however, touch upon the issue. But we restrict our discussion only to the Risk-Metric approach, which postulates conditional variance as a weighted average of past variance and past returns as follows.

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\,r_t^2 = \lambda^t\sigma_0^2 + (1-\lambda)\sum_{k=0}^{t-1}\lambda^k\, r_{t-k}^2 \qquad \text{..... (2)}$$

where σ_t^2 and r_t denote conditional variance and return at time t, respectively; and the parameter λ, known as decay factor, satisfy $0 < \lambda < 1$.

Once we know the estimate of conditional variance at time t, the quantile of conditional distribution (under normality), say $\xi_{p,t}$, can be estimated simply by the Eq.

$$\xi_{p,t} = \mu_t + \tau_p \sigma_t \qquad \text{..... (3)}$$

where, μ_t represents the mean of the conditional distribution.

For daily data, the value of the decay parameter in the Risk Metric approach is generally fixed at λ=0.94 (van den Goorberg and Vlaar, 1999). In our study, we experimented with three alternative values of λ , viz., λ =0.94, 0.96 and 0.98. Thus, as stated earlier, we consider totally five VaR models (two for simple normality and three under Risk-Metric approach with three alternative values of decay parameter) under variance-covariance approach.

2.1.2. Historical Simulation Method

The major advantage with HS approach is that it does not assume any specific form for return distribution, yet captures the characteristics of the price change distribution of the portfolio. Besides, a benefit in using this method is that it can cope with all types of portfolios that are either linear or non-linear. However, accuracy of HS method would depend primarily on the quality/nature of past data. If the past data does not contain highly volatile periods, then HS method would not be able to capture the same. Hence, HS should be applied when we have very large data points that are sufficiently large to take into account all possible cyclical events. HS method takes a portfolio at a point of time and then revalues the same using the historical price series. Once we calculate the daily returns of the price series, then sorting the same in an ascending order and find out the required data point at desired percentiles. Linear interpolation can be used if the required percentile falls in between two data points. The moot question is what length of price series should be used to compute VaR using HS method and what we should do if the price history is not available. It has to be kept in mind that HS method does not allow for time-varying volatility.

2.1.3. Tail-Index Based Methods – Hill's Estimator and VaR Estimation

In financial literature, it is widely believed that high frequency return has fatter tails than can be explained by the normal distribution. The tail-index measures the amount of tail fatness of return distribution and fit within the extreme value theory (EVT). One can therefore, estimate the tail-index and measure VaR based on that. The basic premises of this idea stems from the result that the tails of every fat-tailed distribution converge to the tails of Pareto distribution. The upper tail of such a distribution can be modeled as,

$$\text{Prob}[X > x] \approx C^{\alpha} \, |x|^{-\alpha} \quad (\text{i.e. Prob}[X \leq x] \approx 1 - C^{\alpha} \, |x|^{-\alpha}); \quad x > C \qquad \textbf{..... (4)}$$

Where, C is a threshold above which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

Similarly, lower tail of a fat-tailed distribution can be modeled as

$$\text{Prob}[X > x] \approx 1 - C^{\alpha} \, |x|^{-\alpha} \quad (\text{i.e. Prob}[X \leq x] \approx C^{\alpha} \, |x|^{-\alpha}); \quad x < C \qquad \textbf{..... (5)}$$

Where, C is a threshold below which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

In practice, observations in upper tail of the return distribution are generally positive and those in lower tail are negative. Thus, both of Eq. (4) and Eq. (5) have importance in VaR measurement. The holder of a short financial position suffers a loss when return is positive and therefore, concentrates on upper-tail of return distribution (i.e. Eq. 4) while calculating his VaR (Tsay, 2002, pp. 258). Similarly, the holder of a long financial position would model the lower-tail of return distribution (i.e. use Eq. 5) as a negative return makes him suffer a loss.

From Eq. (4) and (5), it is clear that the estimation of VaR is crucially dependent on the estimation of tail-index α. There are several methods of estimating tail-index and in the present paper, we consider two approaches, viz. (i) Hill's (1975) estimator and (ii) the estimator under ordinary least square (OLS) framework suggested by van den Goorbergh (1999). We consider here the widely used Hill's estimator, a discussion on which is given below.

Hill's Estimator

For given threshold C in right-tail, Hill (1975) introduced a maximum likelihood estimator of $\gamma = 1/\alpha$ as

$$\hat{\gamma} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{X_i}{C}\right) \qquad \textbf{..... (6)}$$

where X_i's, i=1,2, …..,n are n observations (exceeding C) from the right-tail of the distribution.

In practice, however, C is unknown and needs to be estimated. If sample observations come from Pareto distribution, then C would be estimated by the minimum observed value, the minimum order statistic. However, here we are not modeling complete portion of Pareto distribution. We are only dealing with a fat-tailed distribution that has right tail that is approximated by the tail of a Pareto distribution. As a consequence, one has to select a threshold level, say C, above which the Pareto law holds. In practice, Eq. (6) is evaluated based on order statistics in the right-tail and thus, the selection of the order statistics truncation number assumes importance. In other words, one needs to select the number of extreme observations n to operationalise Eq. (6). Mills (1999, pp. 186) discusses a number of available strategies for selecting n and a useful technique for the purpose is due to Phillips, et al. (1996). This approach makes an optimal choice of n that minimises the MSE of the limiting distribution of $\hat{\gamma}$. To implement this strategy, we need estimates of γ for truncation numbers $n_1 = N^{\delta}$ and $n_2 = N^{\tau}$, where $0 < \delta < 2/3 < \tau < 1$. Let $\hat{\gamma}_j$ be the estimate of γ for n $=n_j$, j=1,2. Then the optimal choice for truncation number is $n = [\lambda\, T^{2/3}]$, where λ is estimated as $\hat{\lambda} = |(\hat{\gamma}_1/\sqrt{2})(T/n_2)(\hat{\gamma}_1 - \hat{\gamma}_2)|^{2/3}$. Phillips et al. (1996) recommended setting $\delta = 0.6$ and $\tau = 0.9$ (see Mills, 1999, pp. 186).

Estimating VaR Using Hill's Estimator

Once tail-index α is estimated, the VaR can be estimated as follows (van den Goorbergh and Vlaar, 1999). Let p and q ($p < q$) be two tail probabilities and x_p and x_q are corresponding quantiles. Then $p \approx C^{\alpha} (x_p)^{-\alpha}$ and $q \approx C^{\alpha} (x_p)^{-\alpha}$ indicating that $x_p \approx x_q (q/p)^{1/\alpha}$. Assuming that the threshold in the left-tail of the return distribution corresponds to the m-th order statistics (in assending order), the estimate of x_p be

$$\hat{x}_p = R_{(m)}\left(\frac{m}{np}\right)^{\hat{\gamma}} \qquad \textbf{..... (7)}$$

where $R_{(m)}$ is the m-th order statistics in the assending order of n observations chosen from tail of the underlying distribution; p is the given confidence level for which VaR is being estimated; $\hat{\gamma}$ is the estimate of γ.

The estimate of VaR (with meanings of notations as defined above) would be

$\hat{V}^{p}_{t+1|t}$ = - W_t $\hat{x}_p$; $\hat{x}_p$ is estimate of quantile of return distribution **….(8)**

or

$\hat{V}^{p}_{t+1|t}$ = W_t [1 – exp($\hat{x}_p$)]; $\hat{x}_p$ is estimate of quantile of log-return distribution **….(9)**

The methodology described above estimates tail-index and VaR for right tail of a distribution. To estimate the parameters for left tail, we simply multiply the observations by (–1) and repeat the calculations.

2.2. Use of VaR with one-day Holding Period for Estimating VaR with Longer Holding Period

In practice above methods are used to estimate VaR numbers daily based on one-day holding period returns. However, for computing capital charge, we need the VaR numbers for longer holding period, say 10-days or 30-days. Using the estimates of 1-period VaR, H-period (say, H=10, 30) VaR can be estimated, under certain assumptions, by following approximation (van den Goorbergh and Vlaar, 1999);

$$\text{VaR(H)} \approx \begin{cases} \left(\sqrt[\alpha]{H}\right)\text{VaR}(1) & \text{if VaR(1) is estimated through tail– index } \alpha \\ \left(\sqrt{H}\right)\text{VaR}(1) & \text{for other VaR Models} \end{cases}$$

….. (10)

Where VaR(k) represents the VaR for k-days holding period, k ≥1.

3. VaR Selection/Evaluation Criteria

As stated earlier, banks may be inclined to underestimate their VaRs as this helps in reducing their capital charges. For this reasons, the Basle Committee prescribed certain requirements on VaR models used by banks to ensure their reliability (Wong et al., 2003) as follows;

(1) VaRs must be estimated based on daily return of at least one year with 99% confidence level.
(2) Capital charge is equal to the 60-day moving average of 10-day VaRs multiplied by a factor known as capital multiplier (or simply multiplying factor), or 10-day VaR on the current day, which ever is higher. The multiplying factor may vary from 3 to 4 depending upon the accuracy level of VaR estimates.

In Indian market, RBI has issued guidelines for Primary Dealers (PDs) to use one year and not less than 250 trading days for VaR estimation and the capital charge is prescribed as the higher of (i) the previous day's 99% VaR (for 30-days holding period) measure and (ii) the average of the daily value-at-risk measures on each of the preceding sixty business days, multiplied by a multiplication factor prescribed by RBI (3.30 presently for PDs).

In general, the amount of required capital (RC) for market risk at time (t+1) would be (Lopez, 1998);

$$RC_{t+1} = Max\{VaR_t(H), K * Avg60\_VaR_t(H)\} \quad \textbf{..... (11)}$$

Where H and K represent holding period and multiplying factor (i.e. capital multiplier), respectively, as prescribed by the regulators; $VaR_t(H)$ represents VaR with 99% confidence level for holding period H-days; and $Avg60\_VaR_t(H)$ is the average of $VaR_t(H)$s in last 60-days (i.e. average of estimated VaRs for time points t-59 to t). As stated above, BIS prescribes H=10 days and the value of K is at least 3, which may rise upto 4 depending upon the accuracy of VaR model. However, for PDs in India, RBI prescribes H=30 days and K=3.3.

Further, in order to assess the accuracy of VaR models, the Basle Committee has suggested to conduct backtesting. The basic premises of backtesting stems from that the accuracy of a VaR model can be checked by counting the number of times actual loss of a portfolio exceeds estimated VaR (i.e. VaR estimate fails), say in 100 days. For a VaR with 99% confidence level, logically, one would expect 1 failure in 100 days. But if the number is more (less) than 1%, then the model is under (over) estimating VaR numbers. The Basle Committee provides guidelines for imposing penalty leading to higher multiplication factor, when the number of failure is too high. However, no penalty is imposed when the failure occurs with less frequency than the expected number.

3.1. Backtesting

To do the backtesting, we can think of an indicator variable *I(t)* which is one if return of the day is more than the VaR for the previous day and zero otherwise. Average of the indicator variable should be our VaR percent. Basle Committee (1996b) provides following Backtesting criteria for an internal VaR model (see van den Goorbergh and Vlaar, 1999; Wong et al., 2003, among others)

(1) One-day VaRs are compared with actual one-day trading outcomes.

(2) One-day VaRs are required to be correct on 99% of backtesting days. There should be at least 250 days (around one year) for backtesting.

(3) A VaR model fails in Backtesting when it provides 5% or more incorrect VaRs.

(4) If a bank provides a VaR model that fails in backtesting, it will have its capital multiplier adjusted upward, thus increasing the amount of capital charges.

For carrying out the Backtesting of a VaR model, realized day-to-day returns of the portfolio are compared to the VaR of the portfolio. The number of days when actual portfolio loss was higher that VaR provides an idea about the accuracy of the VaR model. For a good VaR model, this number would approximately be equal to the 1 per cent (i.e. 100 times of VaR probability) of back-test trading days. If the number of violation (i.e. number of days when loss exceeds VaR) is too high, a penalty is imposed by raising the multiplying factor (which is at least 3), resulting in an extra capital charge. The penalty directives provided by the Basle Committee for 250 back-testing trading days is as follows; multiplying factor remains at minimum (i.e. 3) for number of violation upto 4, increases to 3.4 for 5 violations, 3.5 for 6 violations, 3.65 for violations 8, 3.75 for violations 8, 3.85 for violation 9, and reaches at 4.00 for violations above 9 in which case the bank is likely to be obliged to revise its internal model for risk management (van den Goorbergh and Vlaar, 1999).

3.2. Kupiec's Test

The accuracy of a VaR model can also be assessed statistically by applying Kupiec's (1995) test (see, for example, van den Goorbergh and Vlaar, 1999 for an application of the technique). The idea behind this test is that the VaR-violation (i.e. proportion of cases of actual loss exceeding VaR estimate) should be statistically equal to the probability level for which VaR is estimated. Kupiec (1995) proposed a likelihood ratio statistics for testing the said hypothesis.

If z denotes the number of times the portfolio loss is worse than the true VaR in the sample (of size T, say) then z follows a Binomial distribution with parameters (T, p), where p is the probability level of VaR. Note that here z is actually the summation of I_t at T time points. Ideally, the more z/T closes to p, the more accurate estimated VaR is. Thus the null hypothesis $z/T = p$ may be tested against the alternative hypothesis $z/T \neq p$. The likelihood ratio (LR) statistic for testing the null hypothesis against the alternative hypothesis is

$$LR = 2\left[\log_e\left(\left(\frac{z}{T}\right)^z\left(1-\frac{z}{T}\right)^{T-z}\right) - \log_e\left(p^z(1-p)^{T-z}\right)\right] \qquad \textbf{..... (12)}$$

Under the null hypothesis, LR-statistic follows a χ^2-distribution with 1-degree of freedom.

3.3. Tests for Conditional Coverage – Christoffersen's Tests

The VaR estimates are also interval forecasts, which thus, can be evaluated conditionally or unconditionally. While the conditional evaluation considers information available at each time point, the unconditional assessment is made without reference to it. The test proposed by Kupiec provides only an unconditional assessment as it simply counts exceptions (i.e. VaR violations) over the entire backtesting period (Lopez, 1998, 1999). In the presence of time-varying volatility, the conditional accuracy of VaR estimates assumes importance. Any interval forecast ignoring such volatility dynamics may have correct unconditional coverage but at any given time, may have incorrect conditional coverage. In such cases, the Kupiec's test has limited use as it may classify inaccurate VaR as acceptably accurate.

Christoffersen (1998) develops a three step testing procedure: a test for correct unconditional coverage (which is same as Kupiec's test), a test for 'independence', and a test for correct 'conditional coverage' (Sarma, et al., 2003). All these tests use Likelihood-Ratio (LR) statistics. In order to simplify the discussion let us construct an indicator variable as below;

$$I_t = \begin{cases} 1 & \textit{if portfolio loss exceeds VaR at time t} \\ 0 & \textit{otherwise} \end{cases} \qquad \textbf{..... (13)}$$

where the time t varies over the backtesting trading period/days.

In order to have correct conditional coverage, VaR estimates should be such that the series I_t exhibits both unconditional coverage and serial independence. Thus, the test for conditional coverage is actually a joint test of both these two features. The relevant test statistics here is $LR_{cc} = LR_{uc}+LR_{ind}$, where LR_{cc}, LR_{uc} and LR_{ind} represent the likelihood-ratio statistics for testing correct conditional coverage, correct unconditional coverage and independence, respectively. The form of test statistics LR_{uc} is same as Kupiec's test (i.e. LR_{uc}=LR defined in Eq. 12) and the forms of test statistics LR_{cc} and LR_{ind} are as below;

LR Statistics for the test of Independence

$$LR_{ind} = -2\log_e\left[\frac{(1-\hat{\pi}_2)^{(n_{00}+n_{10})}\,\hat{\pi}_2^{(n_{01}+n_{11})}}{(1-\hat{\pi}_{01})^{n_{00}}\,\hat{\pi}_{01}^{n_{01}}\,(1-\hat{\pi}_{11})^{n_{10}}\,\hat{\pi}_{11}^{n_{11}}}\right] \qquad \text{..... (14)}$$

where n_{ij} = number of i values followed by a j in the I_t series, i,j=0,1;

$\pi_{ij} = \Pr\{I_t=i \mid I_{t-1}=j\}$, i,j=0,1;

$\hat{\pi}_{01} = n_{01}/(n_{00}+n_{01})$;

$\hat{\pi}_{11} = n_{11}/(n_{10}+n_{11})$;

$\hat{\pi}_2 = (n_{01}+n_{11})/(n_{00}+n_{01}+n_{10}+n_{11})$.

If the series I_t is serially independence, then LR_{ind} follows a χ^2 distribution with 1 degree of freedom.

LR Statistics for the Test of 'Correct Conditional Coverage'

$$LR_{cc} = -2\log_e\left[\frac{(1-p)^{n_0}\,p^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}\,\hat{\pi}_{01}^{n_{01}}\,(1-\hat{\pi}_{11})^{n_{10}}\,\hat{\pi}_{11}^{n_{11}}}\right] \qquad \text{..... (15)}$$

where n_j=number of j's, j=0,1, in the I_t series; p is the probability level (i.e. tolerance limit) of the VaR estimates, and $\hat{\pi} = n_1/(n_0+n_1)$ is the maximum likelihood estimate of p.

Under the correct conditional coverage, LR_{cc} follows a χ^2 distribution with 2 degrees of freedom.

3.4. Evaluation of VaR Models Using Loss-Function

All the tests mentioned above, ultimately deal with the frequency of the occurrence of VaR violations, either conditional or unconditional, during the backtesting trading days. These tests, however, do not look at the extent/magnitude of additional loss (excess of estimated VaR) at the time of VaR violations/failures. However, a portfolio manager may prefer the case of more frequent but little additional loss than the case of less frequent but huge additional loss. The underlying VaR model in the former case may fail in backtesting but still the total amount of loss (after adjusting for penalty on multiplying factor if any) during the backtesting trading days may be less than that in later case. So long as this is the case, a portfolio manager may even prefer to accept a VaR model even if it fails in backtesting and may be ready to pay penalty (for excess number of VaR violations). This means that the objective function of a portfolio manager is not necessarily be the same as that provided by the backtesting. Each manager may set his own objective function and try to optimize that while managing market risk. But, loss-functions of individual portfolio managers are not available in public domain and thus, it would be impossible to select a VaR model appropriate for all managers. However, discussion on a systematic VaR selection framework by considering a few specific forms of loss-function would provide insight into the issue so as to help individual manager to select a VaR model on the basis of his own loss-function. On this perception, it would be interesting to illustrate the VaR selection framework with the help of some specific forms of loss-function.

The idea of using loss-function for selecting VaR model, perhaps, is proposed first by Lopez (1998, 1999). He shows that the binomial distribution-based test is actually minimizing a typical loss-function – gives score 1 for a VaR exception and a score 0 otherwise. However, it is hard to imagine an economic agent who has such a utility function: one which is neutral to all times with no VaR violation and abruptly shifts to score of 1 in the slightest failure and penalizes all failures equally (Sarma, et al., 2003). Lopez (1998) also considers a more generalised loss-function which can incorporates the regulatory concerns expressed in the multiplying factor and thus is analogous to the adjustment schedule for the multiplying factor for determining required capital. But, he himself see that, like the simple binomial distribution-based loss-function, this loss-function is also based on only the number of exceptions (VaR violations) in backtesting observations – with paying no attention to another concern, the magnitues of loss at the time of failures. In order to handle this situation, Lopez (1998) also proposes a third type of loss-function addressing the magnitude of exception as follows;

$$L_t = \begin{cases} 1 + (Loss_t - VaR_{t-1})^2 & if\ Loss_t > VaR_{t-1} \\ 0 & otherwise \end{cases} \qquad \text{..... (16)}$$

where $Loss_t$ and VaR_t, respectively, are the magnitude/amount of loss and estimated Value-at-Risk at time t. L_t denotes the score in loss-function at time t.

In the spirit of Lopez (1998), Sarma et al. (2003) consider two loss-functions, viz., regulatory loss function and the firm's loss function, as follows;

Regulatory Loss Function

$$L_t = \begin{cases} (Loss_t - VaR_{t-1})^2 & if\ Loss_t > VaR_{t-1} \\ 0 & otherwise \end{cases} \qquad \text{..... (17)}$$

Firm's Loss Function

$$L_t = \begin{cases} (Loss_t - VaR_{t-1})^2 & if\ Loss_t > VaR_{t-1} \\ \alpha\ VaR_{t-1} & otherwise \end{cases} \qquad \text{..... (18)}$$

where α represents the opportunity cost of capital.

As seen above, loss-function can be formed in several ways, and portfolio managers have to form their own loss-functions depending upon their risk-management and investment strategy. In this empirical study, what we are presenting is a framework for using loss-function in selecting VaR models. In so doing, we implement two loss-function given by Eq.s (16) & (17)[3]. It may be noted that if the number of VaR violations (i.e. loss exceeds estimated VaR) in backtesting is X, then the value of loss-function in Eq. (16) becomes [X + Σ_t {I_t ($Loss_t$-VaR_{t-1})^2}] and that for loss-function in Eqn. (17) becomes [Σ_t {I_t ($Loss_t$-VaR_{t-1})^2}], where Σ_t indicates the summation over all times t in backtesting trading days. While Eq. (16) represents a loss-function that penalizes for both frequency of VaR violation as well as magnitude of loss above estimated VaR, the loss function in Eq. (17) considers only the magnitude of excess loss. Thus, someone who wants a loss-function that penalizes a VaR model more severely that binomial loss function (i.e. backtesting), may prefer the loss-function Eq. (16) over that in Eq. (17). This is because Σ_t {I_t ($Loss_t$-VaR_{t-1})^2}] need not always be more than X.

The basic reason for not considering loss-function in Eq. (18) in our empirical study is that the required capital for market risk is charged irrespective of the fact whether loss in the underlying portfolio occurs or not. Thus, it is not clear as to why the opportunity cost of holding capital is considered only when loss does not exceed estimated VaR. Besides, when loss-occurs, the score of loss-function depends only on the amount of loss above estimated VaR and does not penalize the VaR model for high occurrences of VaR violations. As discussed above, a conservative portfolio manager who want to penalize a VaR model for VaR violations more severely than binomial loss-function, may not prefer this type of formulation of loss-function.

4. Data

The database used in this empirical study is same as Nath and Samanta (2003) and consists of information on select GOI bonds and two representative portfolios, one for banks and another for Primary Dealers (PDs), of GOI bonds. The selected 31 bonds come from both illiquid as well as liquid basket. Liquid bonds are those bonds where we observe trading regularly while illiquid bonds are infrequently traded. For applying uniform market conventions in valuation and simplicity, we have not taken T-Bills in our portfolio but it can be included as well. In this process, we consider 31 GOI bonds for our study. Two representative portfolios of GOI bonds considered here have been constructed keeping in mind Banks and PDs. For the representative portfolio for entire banking sector that hold almost all securities issued by the Govt., we assign weight to each bond proportional to its outstanding issue size. For the representative portfolio for PDs, we assume that they hold all liquid bonds.

4.1. Valuation of Individual GOI Bonds

In order to estimate VaR for a portfolio of GOI bonds, one has to derive the historical data on price/return of the portfolio. Construction of such historical series is difficult because bonds change their values everyday (assuming *ceteris paribus*) because of time to maturity come down everyday and hence the Present-Value (PV) changes. As well known, derivation of PV of bonds requires information on zero coupon yield curve (ZCYC). But the main difficulty for estimating ZCYC is that zero-coupon bonds are generally not traded for all maturity periods (particularly for medium-to-long term maturities). So, one has to extract ZCYC from the traded data/prices of coupon-bearing bonds. There exists a vast literature on the subject (see for instance,

[3] While saying this, one may note that we are also using the binomial loss-function, which is implicitly considered in backtesting.

McCulloch, 1971; Nelson and Siegel, 1987; Vasicek and Fong, 1982; Shea, 1985; among others), though consensus has not yet emerged on choosing the best technique. It is seen that no single technique can be the best for all economies and all times. While choosing a technique for estimating ZCYC, one has to consider several factors, such as, market structure and depth, degree of liquidity of different bonds. The estimation of ZCYC for GOI bonds, however, does not fall under the agenda of this study and we have made use of the ZCYC estimated by the National Stock Exchange (NSE). The NSE uses the methodology proposed by Nelson and Siegel (1987) and has created database on estimated ZCYC (and parameters of Nelson-Siegel functional form) from January 1997. These estimates are available in public domain free of cost. Accordingly, we use NSE ZCYC parameters to price the individual GOI bonds and portfolios of such bonds from 1997. Our data period ends on June 23, 2003. Based on the estimated historical prices of a bond, return series is derived as

$$R_t = 100 * \{ \log_e(P_t) - \log_e(P_{t-1})\} \qquad \text{..... (19)}$$

Where P_t denotes the price of the bond at time t.

However, there was an apparent estimation problem on 23-05-1997 when suddenly the yields had dropped abnormally and increased abnormally next day. To ensure proper use of data, we had looked at the underlying market and did not observe any abnormal trading behaviour. Hence while using the data we considered the data point of 23-05-1997 as an estimation problem and replaced the same with the average value of previous day's and next day's model prices and calculated returns accordingly. The returns considered in this study are the continuously compounded returns, which are derived as the first difference of daily observations on logarithm of prices. In this process, we get 1874 daily time series observations on return on each of 31 selected GOI bonds.

4.2. Two Hypothetical Portfolios of GOI Fixed Income Securities

Estimation of VaR for an individual bond can simply be calculated using the return series implied by the historical price of the bond constructed using the NSE ZCYC. But for calculating VaR of a portfolio, we need to find out the single price series of the portfolio using the historical yield curves. In our portfolio of GOI bonds, weight of a bond is equal to its share in the total portfolio value. While constructing the historical price series of the portfolio, it is also assumed that weights have remained as it is today. This is because VaR envisages to find out the possible maximum loss of a portfolio today in a given time horizon with a level of probability and hence the portfolio needs to be maintained as of today's composition.

The portfolio consists of many securities and in our case we are concerned with only Gilts. The basic price Eq. of the portfolio (of n bonds) can be written as follows:

$$\Pr_{portfolio} = w_1 * \Pr_{bond1} + w_2 * \Pr_{bond2} + ... + w_n * \Pr_{bondn} \qquad \text{..... (20)}$$

where Pr denotes the price of bond/portfolio at time t and w_i, $i=1,2,\ldots,n$ denotes the proportionate value of the holding of security i at the end of day *t*.

5. Empirical Results

5.1. Estimated VaRs from Competing Models for Calculating Required Capital

In this section we report estimated VaR figures (in percentage terms) for two alternative holding periods (H), viz.., H=10 days and 30 days. These VaRs with holding period H-days are derived indirectly based on estimated one-day VaRs using Eq. (10). The VaRs for one-day holding period are calculated directly from daily returns. As stated earlier, all VaR estimates correspond to the 99% confidence level (i.e. probability level 0.01) and relate to the left-tail of return distribution. Relevant results for two representative portfolios of GOI bonds are given in Table 1 and corresponding results for selected 31 GOI bonds are given in Annexure 1. In Table 1 and Annexure 1, we use the symbol VaR(H), to denote the VaR with H-days holding period for the last day in our database and Avg60_VaR(H) to denote the average of VaRs with H-days holding period in last 60 days, where holding period H is either 10-days or 30-days. One can put these estimates (i.e. VaR(H) and Avg60_VaR(H)) in Eq. (11) to derive the required capital for a given multiplying factor.

As stated above, an important point needs to be noted here is that all VaR estimates provided in Table 1 and Annexure 1 are in percentage form, and thus, may actually be termed as the relative VaR (Wong, et al., 2003), which refers to the percentage of a portfolio value which may be lost after h-holding period with a specified probability (i.e. the probability level of VaR). The absolute VaR (i.e. the VaR expressed in Rupees term) can easily be computed by multiplying the portfolio values with the estimated relative VaR. Similarly, the capital charge (Eq. 11) can also be represented in two alternative forms, viz., relative (i.e. in percentage) or absolute (i.e. in rupees terms). The additional information we require to convert a relative VaR/capital charge in a day to a corresponding absolute term (i.e. rupees term) figures is the value of the portfolio. For example, from Table 1 we see that the VaR(10) for the representative portfolio (of GOI Bonds) for PDs in the last day in our data period (i.e. June 23, 2003) has been estimated at 3.9039 % under normal method using full sample. Thus, if value of the portfolio at that day was Rs. 1000, estimated absolute VaR(10) in Rupees terms would be Rs. 39.039 (i.e. 3.9039 x 10). = 39.039 using the normal method (full sample).

Table 1: Estimated VaRs for Two Representative Portfolios of GOI Bond

		Variance-Covariance (Normal) Method					Historical Simulation		Tail-Index (Hill's Estimator)	
	Description of Estimate (in per cent)	Simple (homoscadastic)		Risk Metric with λ (conditional heteroscadasctic)						
Portfolio		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
PDs	VaR (10)	3.9039	2.9201	2.5269	1.5993	1.0647	4.6503	3.8809	4.6719	9.8598
	Avg60_VaR(10)	3.9338	3.0510	3.3990	2.8738	2.3285	4.6682	3.9264	4.7518	7.2682
	VaR (30)	6.7617	5.0577	4.3767	2.7701	1.8442	8.0546	6.7219	8.0066	21.9407
	Avg60_VaR(30)	6.8136	5.2845	5.8872	4.9775	4.0331	8.0855	6.8008	8.1652	14.6022
Banks	VaR (10)	3.4145	2.6237	2.3002	1.4538	0.9544	4.0668	3.2897	4.0805	6.8948
	Avg60_VaR(10)	3.4406	2.7601	3.0926	2.6206	2.1270	4.0755	3.3844	4.1375	7.1701
	VaR (30)	5.9142	4.5444	3.9840	2.5181	1.6531	7.0439	5.6979	7.0087	14.2514
	Avg60_VaR(30)	5.9593	4.7807	5.3565	4.5390	3.6840	7.0590	5.8619	7.1136	14.6526

The columns in Table 1 and Annexure 1 are self-explanatory. As can be seen therein, we present estimated VaRs from five alternative schemes under normal method (one for full sample estimate, one for rolling sample estimate, and three for Risk Metric approach corresponding to three alternative decay factors, λ = 0.98, 0.96 and 0.94) and full sample as well as rolling sample estimates under each of Historical Simulation and Tail-Index (Hill's estimator) based approach. Thus, we have considered nine competing VaR models/strategies. Full sample estimates at any day, say t, are derived based on all returns from day 1 to t. In the case of rolling sample estimates, we fix the size/length of the rolling windows at 500 days[4]. So, for rolling sample estimates of VaRs at time t, returns for time points (t-499) to t are used. The columns with titles 'Full' and 'Rolling' provide estimates corresponding to full sample and rolling sample, respectively.

5.2. Evaluating Competing VaR Models

For evaluating performance of competing VaR models, we first carried out the backtesting with daily returns for last 290 days (covering about a period of one year as backtesting observations) in our sample. The backtesting strategy adopted for rolling sample estimate is as follows; estimate 1-day VaR for t-th day based on returns for time points (t-499)-th to t-th day. If the return for the (t+1)-th day is worse than VaR (i.e. for left tail, if percentage return is lower than negative of Var expressed in terms of percentage), we say a failure occurs. Then estimate 1-day VaR for (t+1)-th day and compare the same with (t+2)-th day's return to see whether another failure occurs or not. The process is repeated till the return for the last day in our database is compared with previous day's VaR estimates. In our empirical exercise, t is so chosen that last 290 observations on return are compared with previous day's VaR estimates. In the case of full sample case, the backtesting is similarly performed except that at any time point t, VaR estimates are obtained based on all returns available upto time t (i.e. returns for time points 1 to t).

As our VaR estimates have probability level 0.01 and the Backtesting trading days cover 290 daily returns, expected number of failures for a good VaR model (i.e. the number of occasions out of 290 days when actual return is worse than VaR) is 3. In *Table 2*, we report the results of Backtesting for two hypothetical portfolios. Detailed bond-wise results of Backtesting are presented in *Annexure 2*. Our empirical results show that for certain VaR models, backtesting with respect to many bonds/portfolios shows no VaR violations (i.e. no return were worse than VaR estimates). In such cases, it would be difficult to test the unconditional as well as conditional coverage. This is because in such a case some of the required probability estimates would be zero and so the LR test statistics in Eqs. 13-15 would be undefined. Therefore, we do not conduct statistical tests for unconditional/conditional coverage. However, loss-function based evaluation of competing VaR models is done for each selected bond/portfolio. Relevant results are also presented in Table 2 (for selected portfolios) and Annexure 2 (for selected GOI bonds).

In Table 2 and Annexure 2, there are three rows under each portfolio/bond. The first row indicates number of VaR violations (i.e. failures) in 290-days backtesting observations. In second row, denoted by 'Loss 1', represents the estimated value of the loss-function given in Eq. (17) over 290 backtesting observations. Finally, the third row, indicated by 'Loss2', presents the value of the loss-function in Eq. (16). Thus, by construction, value of 'Loss2' would be equal to the sum of the score in backtesting

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[4] As per the BIS guidelines, VaR should be estimated using daily returns for at least one year. This requirement is satisfied for 500-days rolling sample size. However, we do admit that the length of rolling window chosen here is arbitrary and one may experiment with other alternative rolling sample sizes, say, 750, 1000, 1250, 1500, so on.

(i.e. number of failures in 290 days backtesting observations) and the estimated value of 'Loss1'.

Table 2: Assessing Competing VaR Models for Two Portfolios

Portfolio	Evaluation Criterion/Function	Variance-Covariance (Normal) Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Simple (homoscadastic)		Risk Metric with λ (conditional heteroscadasctic)						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
PDs	Backtesting	2	8	7	8	7	2	3	0	0
	Loss 1	0.1336	1.0555	0.9383	0.5154	0.3286	0.0031	0.1664	0	0
	Loss 2	2.1336	9.0555	7.9383	8.5154	7.3286	2.0031	3.1664	0	0
Banks	Backtesting	2	8	7	7	7	2	3	2	0
	Loss 1	0.1856	0.8681	0.8321	0.4347	0.2382	0.0282	0.2488	0.0253	0
	Loss 2	2.1856	8.8681	7.8321	7.4347	7.2382	2.0282	3.2488	2.0253	0

Our empirical results reveal that in generally normal-based methods/strategies (including Risk-Metric approach) are associated with too many failures over backtesting observations. This is reflection of the under estimated VaR. On the other hand, historical simulation performs very well in backtesting, providing number of failures very close to expectation (i.e. 3), though at times full sample VaR estimates are quite conservative as reflected in zero score in backtesting. Then comes the VaR model using Hill's estimator. It appears that this approach provides too conservative VaR estimates. Because, the number of failures in backtesting for this approach is almost uniformly lower than that of HS method. Besides, the number of failures for Hill's estimator based approach never exceeds the expected number 3. The evaluation of the competing VaR models with the help of loss-functions gives no different conclusions. Interesting point, however, is that the approach based on Hill's estimator provides conservative VaR estimates and generally provides the least magnitude of VaR violations (as captured through loss-functions).

6. Concluding Remarks

This paper has evaluated a number of available VaR models, such as, variance-covariance/normal (including Risk-Metric approach), historical simulation and tail-index based method for estimating VaR for a number of selected GOI bonds and representative portfolios of GOI bonds for banks and PDs. We discuss several evaluation criteria, such as, backtesting, statistical tests for accuracy in VaR estimates and loss-functions based assessment. Empirical results, however, are presented only for the backtesting and loss-function based evaluation. Statistical tests are not conducted for certain practical reasons stated earlier. Our empirical results are interesting. It is seen that normal methods (including Risk-Metric approach) generally under-estimate VaRs. On the other hand, VaR models based on HS and tail-index (using Hill's estimator) are quite good, though the later produces slightly more conservative VaR estimates. But when we look at the available loss-functions, tail-index method appears to give least magnitude of excess loss (i.e. loss over estimated VaR). These results, however, are tentative. One needs to experiment with alternative sizes of rolling sample to check the robustness of the results. Moreover, it would be interesting to search for more appropriate loss-functions while evaluating VaR models.

Annexure 1: Estimated VaRs for Each Selected GOI Bond

GOI Bond	Description of Estimate	*Normal Method*					Historical Simulation		Tail-Index (Hill's Estimator)	
		Static Model		Risk Metric with λ						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2004-12.50%	VaR (10)	1.5194	1.3763	1.2438	1.0148	0.8757	1.9577	1.7830	1.8746	1.9243
	Avg60_VaR(10)	1.5272	1.4640	1.4925	1.3920	1.2574	1.9663	1.8801	1.8985	2.2997
	VaR (30)	2.6317	2.3838	2.1543	1.7577	1.5168	3.3908	3.0883	3.0645	3.3717
	Avg60_VaR(30)	2.6451	2.5358	2.5851	2.4110	2.1779	3.4057	3.2564	3.1070	4.1391
2005 11.19%	VaR (10)	2.2193	2.2137	2.1869	1.7640	1.5121	2.8697	2.3981	2.7453	2.2003
	Avg60_VaR(10)	2.2282	2.3451	2.6418	2.4442	2.1912	2.8726	2.5035	2.8147	2.7383
	VaR (30)	3.8439	3.8342	3.7878	3.0553	2.6191	4.9705	4.1536	4.5957	3.4575
	Avg60_VaR(30)	3.8593	4.0619	4.5758	4.2335	3.7953	4.9755	4.3361	4.7440	4.5038
2006-11.68%	VaR (10)	2.3228	2.3521	2.2959	1.8303	1.5563	2.8776	2.6136	3.0929	2.5486
	Avg60_VaR(10)	2.3325	2.5200	2.7923	2.5683	2.2899	2.9345	2.7748	3.092	2.9406
	VaR (30)	4.0232	4.0740	3.9766	3.1702	2.6956	4.9841	4.5269	5.3536	4.1228
	Avg60_VaR(30)	4.0400	4.3647	4.8364	4.4484	3.9662	5.0826	4.8060	5.3440	4.8588
2007 11.90%	VaR (10)	2.4828	2.4428	2.2857	1.7678	1.4680	3.0260	3.1002	3.18291	3.4061
	Avg60_VaR(10)	2.4954	2.6495	2.8263	2.5662	2.2599	3.0531	3.6106	3.4295	4.0010
	VaR (30)	4.3004	4.2311	3.9589	3.0619	2.5426	5.2412	5.3697	5.5396	5.9559
	Avg60_VaR(30)	4.3222	4.5891	4.8953	4.4448	3.9143	5.2882	6.2538	6.1125	7.0539
2008 11.50%	VaR (10)	4.7743	3.3079	2.8044	1.8254	1.2707	5.5596	4.3848	6.1370	6.0420
	Avg60_VaR(10)	4.8107	3.4321	3.7403	3.1915	2.6289	5.5632	4.5334	6.0341	6.7164
	VaR (30)	8.2693	5.7294	4.8573	3.1617	2.2009	9.6296	7.5946	10.8698	11.266
	Avg60_VaR(30)	8.3323	5.9446	6.4783	5.5279	4.5534	9.6358	7.8521	10.6022	12.6572
2008 12%	VaR (10)	3.6735	2.8549	2.3265	1.5651	1.1231	3.9501	3.1308	3.2092	2.7734
	Avg60_VaR(10)	3.7005	3.0233	3.0597	2.6474	2.2181	3.9646	3.5568	3.286	3.1412
	VaR (30)	6.3626	4.9448	4.0297	2.7108	1.9453	6.8418	5.4228	5.5316	4.4494
	Avg60_VaR(30)	6.4095	5.2365	5.2995	4.5854	3.8418	6.8669	6.1606	5.6899	5.1171
2009 6.96%	VaR (10)	3.1892	2.7818	2.4230	1.7801	1.4089	4.1159	3.2203	3.9371	3.3824
	Avg60_VaR(10)	3.2108	2.9973	3.0665	2.7298	2.3646	4.1245	3.9764	4.0421	3.6207
	VaR (30)	5.5239	4.8182	4.1968	3.0833	2.4403	7.1289	5.5776	6.9354	5.6054
	Avg60_VaR(30)	5.5613	5.1915	5.3114	4.7281	4.0957	7.1439	6.8874	7.1482	5.9608
2009-11.99%	VaR (10)	2.9198	2.6044	2.1935	1.5836	1.2326	3.7642	3.1607	3.7567	3.5351
	Avg60_VaR(10)	2.9391	2.8120	2.8028	2.4825	2.1329	3.8682	3.9053	3.6899	3.413
	VaR (30)	5.0573	4.5110	3.7993	2.7428	2.1349	6.5198	5.4746	6.6793	6.0138
	Avg60_VaR(30)	5.0906	4.8706	4.8545	4.2998	3.6943	6.6999	6.7642	6.5206	5.6288

Annexure 1: Estimated VaRs for Each Selected GOI Bond (*Contd.*)

GOI Bond	Description of Estimate	*Normal Method*					Historical Simulation		Tail-Index (Hill's Estimator)	
		Static Model		Risk Metric with λ						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2010 6.20%	VaR (10)	3.5892	2.9521	2.3287	1.6229	1.2134	4.1152	3.3890	4.0190	3.0293
	Avg60_VaR(10)	3.6149	3.1615	3.0200	2.6468	2.2513	4.1358	3.7028	4.0763	3.6073
	VaR (30)	6.2167	5.1133	4.0335	2.8110	2.1017	7.1278	5.8700	6.8616	4.7467
	Avg60_VaR(30)	6.2611	5.4759	5.2308	4.5845	3.8994	7.1634	6.4134	6.9831	5.8490
2010 7.55%	VaR (10)	3.5677	2.9138	2.5029	1.7921	1.3797	4.1381	3.3006	3.9494	3.0941
	Avg60_VaR(10)	3.5933	3.1077	3.1993	2.8216	2.4228	4.1437	3.6728	3.95371	3.5031
	VaR (30)	6.1794	5.0468	4.3352	3.1040	2.3897	7.1674	5.7169	6.7191	4.9061
	Avg60_VaR(30)	6.2237	5.3827	5.5413	4.8872	4.1964	7.1772	6.3615	6.6992	5.6662
2011 9.39%	VaR (10)	3.7982	2.9455	2.3688	1.6033	1.1592	3.9915	3.2092	4.2265	4.0205
	Avg60_VaR(10)	3.8261	3.1200	3.1084	2.6959	2.2663	4.0172	3.6173	4.2587	3.2776
	VaR (30)	6.5787	5.1017	4.1030	2.7770	2.0078	6.9135	5.5585	7.2220	6.8303
	Avg60_VaR(30)	6.6270	5.4040	5.3839	4.6694	3.9253	6.9579	6.2654	7.2766	5.1746
2011A 11.50%	VaR (10)	3.7405	2.8847	2.3500	1.5788	1.1314	3.9857	3.1776	4.3548	3.9016
	Avg60_VaR(10)	3.7681	3.0516	3.0923	2.6744	2.2399	3.9946	3.6258	4.4226	3.5770
	VaR (30)	6.4788	4.9964	4.0703	2.7346	1.9597	6.9034	5.5037	7.5486	6.6288
	Avg60_VaR(30)	6.5266	5.2855	5.3560	4.6322	3.8797	6.9189	6.2801	7.6748	5.8282
2012 6.85%	VaR (10)	4.3981	3.2136	2.5981	1.7500	1.2616	4.5264	3.6312	4.7759	4.4967
	Avg60_VaR(10)	4.4308	3.3736	3.4181	2.9581	2.4831	4.5300	3.8402	4.8183	4.4512
	VaR (30)	7.6177	5.5662	4.5000	3.0311	2.1852	7.8400	6.2893	8.0823	7.7432
	Avg60_VaR(30)	7.6744	5.8432	5.9202	5.1235	4.3009	7.8461	6.6515	8.1589	7.5046
2012 7.40%	VaR (10)	4.3426	3.1782	2.5783	1.7330	1.2461	4.4703	3.6071	4.7236	4.2747
	Avg60_VaR(10)	4.3750	3.3366	3.3947	2.9356	2.4615	4.4786	3.8479	4.7977	4.5328
	VaR (30)	7.5216	5.5048	4.4658	3.0017	2.1583	7.7427	6.2477	7.9999	7.3008
	Avg60_VaR(30)	7.5777	5.7791	5.8798	5.0846	4.2635	7.7571	6.6648	8.1501	7.7272
2012 - 11.03%	VaR (10)	3.9825	2.9727	2.4439	1.6286	1.1569	4.1693	3.3796	4.6142	4.2652
	Avg60_VaR(10)	4.0121	3.1289	3.2269	2.7823	2.3223	4.1994	3.6403	4.7073	4.4565
	VaR (30)	6.8978	5.1489	4.2330	2.8209	2.0038	7.2214	5.8536	7.9721	7.3892
	Avg60_VaR(30)	6.9492	5.4194	5.5891	4.8191	4.0224	7.2736	6.3051	8.1576	7.7169
2013-7.27%	VaR (10)	4.8666	3.3774	2.8123	1.8747	1.3422	5.2831	4.2176	6.1261	6.1055
	Avg60_VaR(10)	4.9033	3.5124	3.7181	3.2010	2.6706	5.3066	4.3181	5.6303	4.6318
	VaR (30)	8.4293	5.8499	4.8711	3.2471	2.3247	9.1507	7.3052	10.8003	11.2278
	Avg60_VaR(30)	8.4927	6.0837	6.4400	5.5443	4.6256	9.1913	7.4791	9.6759	7.7979

Annexure 1: Estimated VaRs for Each Selected GOI Bond (*Contd.*)

GOI Bond	Description of Estimate	Normal Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Static Model		Risk Metric with λ						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2013 9.81%	VaR (10)	4.3888	3.1414	2.6160	1.7330	1.2262	4.6698	3.8158	5.1347	4.1109
	Avg60_VaR(10)	4.4218	3.2836	3.4641	2.9784	2.4785	4.6731	4.0629	5.3153	4.3434
	VaR (30)	7.6016	5.4411	4.5311	3.0016	2.1238	8.0884	6.6092	8.8761	6.8648
	Avg60_VaR(30)	7.6587	5.6873	5.9999	5.1587	4.2928	8.0941	7.0371	9.2384	7.3387
2014 6.72%	VaR (10)	4.5322	3.2638	2.6549	1.7859	1.2870	4.6691	3.7726	5.2517	4.6733
	Avg60_VaR(10)	4.5660	3.4178	3.4955	3.0227	2.5355	4.7182	3.9032	5.3034	4.4523
	VaR (30)	7.8500	5.6530	4.5984	3.0932	2.2292	8.0872	6.5343	9.1017	8.1490
	Avg60_VaR(30)	7.9086	5.9199	6.0544	5.2355	4.3915	8.1721	6.7605	9.1965	7.5009
2014 7.37%	VaR (10)	3.4659	7.1298	2.9073	1.9304	1.3793	5.8036	4.4572	6.4296	6.2890
	Avg60_VaR(10)	3.5928	7.1855	3.8509	3.3083	2.7528	5.8133	4.5146	6.5651	5.2641
	VaR (30)	6.0030	12.3492	5.0356	3.3435	2.3891	10.0521	7.7200	11.4319	11.6123
	Avg60_VaR(30)	6.2230	12.4456	6.6700	5.7301	4.7679	10.0689	7.8196	11.7124	9.1267
2015 9.85%	VaR (10)	5.1105	3.4905	2.9478	1.9226	1.3480	6.0114	4.7073	6.3557	5.8926
	Avg60_VaR(10)	5.1496	3.6093	3.9309	3.3529	2.7619	6.0252	4.7928	6.2847	5.5701
	VaR (30)	8.8517	6.0457	5.1057	3.3301	2.3349	10.4120	8.1533	11.0873	10.5706
	Avg60_VaR(30)	8.9193	6.2514	6.8086	5.8075	4.7838	10.4360	8.3014	10.915	9.7678
2015 10.47%	VaR (10)	4.8243	3.3332	2.8210	1.8440	1.2910	5.5779	4.3903	6.2454	7.0777
	Avg60_VaR(10)	4.8610	3.4578	3.7569	3.2104	2.6502	5.5905	4.5402	6.2759	6.5534
	VaR (30)	8.3559	5.7732	4.8860	3.1939	2.2360	9.6612	7.6041	11.1295	13.6973
	Avg60_VaR(30)	8.4195	5.9890	6.5072	5.5605	4.5903	9.6830	7.8639	11.1604	12.1484
2016 10.71%	VaR (10)	5.1249	3.5096	2.9604	1.9213	1.3400	6.0654	4.7814	6.0493	5.9644
	Avg60_VaR(10)	5.1641	3.6271	3.9551	3.3666	2.7650	6.0905	4.9037	6.1039	5.3444
	VaR (30)	8.8765	6.0788	5.1276	3.3278	2.3210	10.5055	8.2816	10.3676	10.6666
	Avg60_VaR(30)	8.9446	6.2823	6.8504	5.8312	4.7891	10.5490	8.4934	10.4672	9.2497
2017 7.46%	VaR (10)	6.1258	4.1408	3.3826	2.2127	1.5849	7.8939	5.5677	7.5945	6.6535
	Avg60_VaR(10)	6.1729	4.2481	4.5133	3.8341	3.1472	7.9041	5.5719	7.6538	6.5342
	VaR (30)	10.6102	7.1721	5.8589	3.8324	2.7452	13.6726	9.6435	13.1105	11.8431
	Avg60_VaR(30)	10.6917	7.3580	7.8173	6.6408	5.4511	13.6902	9.6509	13.2194	11.4275
2017 8.07%	VaR (10)	5.7998	3.9136	3.2430	2.1183	1.5052	7.1834	5.3096	6.6504	5.14904
	Avg60_VaR(10)	5.8443	4.0235	4.3272	3.6814	3.0255	7.2217	5.3898	6.7498	5.6119
	VaR (30)	10.0455	6.7786	5.6170	3.6691	2.6071	12.4420	9.1964	11.2078	8.6016
	Avg60_VaR(30)	10.1226	6.9689	7.4949	6.3763	5.2403	12.5084	9.3354	11.3922	9.5526

Annexure 1: Estimated VaRs for Each Selected GOI Bond (*Concld.*)

GOI Bond	Description of Estimate	*Normal Method*					Historical Simulation		Tail-Index (Hill's Estimator)	
		Static Model		Risk Metric with λ						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2018 6.25%	VaR (10)	6.5942	4.4451	3.5716	2.3524	1.7106	8.6838	5.9028	8.1219	6.4938
	Avg60_VaR(10)	6.6449	4.5502	4.7564	4.0411	3.3221	8.7278	5.9302	8.3069	6.5831
	VaR (30)	11.4215	7.6991	6.1862	4.0744	2.9629	15.0409	10.2240	13.9747	11.1076
	Avg60_VaR(30)	11.5093	7.8813	8.2384	6.9994	5.7540	15.1170	10.2713	14.3697	11.2628
2019 10.03%	VaR (10)	6.0358	4.2403	3.4027	2.2174	1.5962	7.8934	5.8531	7.3865	6.3055
	Avg60_VaR(10)	6.0823	4.3470	4.5468	3.8431	3.1350	7.9196	5.8821	7.5302	5.9434
	VaR (30)	10.4543	7.3444	5.8936	3.8406	2.7646	13.6718	10.1379	12.6519	10.8219
	Avg60_VaR(30)	10.5348	7.5293	7.8753	6.6564	5.4299	13.7171	10.1881	12.9375	10.0379
2020 10.70%	VaR (10)	6.0702	4.3217	3.4467	2.2557	1.6401	7.8923	5.9298	7.8312	5.4687
	Avg60_VaR(10)	6.1168	4.4274	4.5994	3.8852	3.1691	7.9463	5.9669	7.8989	5.9006
	VaR (30)	10.5138	7.4854	5.9699	3.9070	2.8407	13.6699	10.2707	13.6246	9.0112
	Avg60_VaR(30)	10.5946	7.6684	7.9663	6.7293	5.4891	13.7634	10.3349	13.7637	9.8972
2022 8.35%	VaR (10)	6.9245	5.1875	3.9977	2.7537	2.1710	9.0381	6.6782	8.8474	8.0445
	Avg60_VaR(10)	6.9763	5.2830	5.2397	4.4454	3.6731	9.1204	6.6782	8.9361	8.7287
	VaR (30)	11.9936	8.9851	6.9243	4.7695	3.7604	15.6545	11.5669	15.2464	14.2216
	Avg60_VaR(30)	12.0832	9.1505	9.0755	7.6997	6.3619	15.7970	11.5669	15.4075	15.6829
2023 6.30%	VaR (10)	7.7308	5.9964	4.5473	3.2719	2.7158	10.1845	7.3682	10.0521	11.0038
	Avg60_VaR(10)	7.7869	6.0837	5.8552	4.9985	4.1856	10.3568	7.3682	10.1607	9.4047
	VaR (30)	13.3901	10.3861	7.8761	5.6670	4.7039	17.6401	12.7622	17.3409	20.1467
	Avg60_VaR(30)	13.4873	10.5373	10.1415	8.6577	7.2496	17.9384	12.7622	17.5334	16.4888
2026 10.18%	VaR (10)	7.1630	5.9499	4.8100	3.6682	3.1975	9.1820	7.6681	9.3712	9.9552
	Avg60_VaR(10)	7.2106	6.0141	6.0234	5.2130	4.4476	9.2310	7.6681	9.4377	8.2623
	VaR (30)	12.4067	10.3055	8.3312	6.3535	5.5383	15.9037	13.2815	16.1730	17.9358
	Avg60_VaR(30)	12.4892	10.4166	10.4328	9.0291	7.7035	15.9885	13.2815	16.2798	14.0123
2032 7.95%	VaR (10)	8.1548	7.9681	7.4611	6.5049	6.1632	11.0001	8.9494	11.1956	9.7387
	Avg60_VaR(10)	8.1839	7.9269	8.5756	7.7365	6.9370	11.0707	8.9494	11.5026	11.0375
	VaR (30)	14.1245	13.8012	12.9230	11.2668	10.6751	19.0528	15.5008	19.5351	16.0063
	Avg60_VaR(30)	14.1750	13.7298	14.8534	13.4000	12.0152	19.1750	15.5008	20.1867	18.7928

Annexure 2: Evaluating Competing VaR Models for Each Selected GOI Bond

GOI Bond	Description of Estimate	Normal Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Static Model		Risk Metric with λ						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2004-12.50%	Backtesting	3	3	6	5	5	1	2	1	1
	Loss 1	0.1875	0.1996	0.4380	0.4184	0.4049	0.0732	0.0892	0.0478	0.0487
	Loss 2	3.1875	3.1996	6.4380	5.4184	5.4049	1.0732	2.0892	1.0478	1.0487
2005 11.19%	Backtesting	4	4	6	7	6	2	3	2	2
	Loss 1	1.0013	1.0110	1.2892	1.1787	1.1242	0.5056	0.8195	0.4489	0.6339
	Loss 2	5.0013	5.0110	7.2892	8.1787	7.1242	2.5056	3.8195	2.4489	2.6339
2006-11.68%	Backtesting	4	3	6	6	6	2	2	2	2
	Loss 1	1.1005	1.0591	1.3122	1.1778	1.1068	0.6622	0.7588	0.5311	0.5292
	Loss 2	5.1005	4.0591	7.3122	7.1778	7.1068	2.6622	2.7588	2.5311	2.5292
2007 11.90%	Backtesting	4	4	7	8	6	2	2	2	2
	Loss 1	0.8474	0.8350	1.0239	0.8916	0.8108	0.5194	0.3676	0.3791	0.2421
	Loss 2	4.8474	4.8350	8.0239	8.8916	6.8108	2.5194	2.3676	2.3791	2.2421
2008 11.50%	Backtesting	2	7	6	6	6	0	3	0	1
	Loss 1	0.0271	1.1649	1.0182	0.5935	0.3724	0.0000	0.1908	0.0000	0.0836
	Loss 2	2.0271	8.1649	7.0182	6.5935	6.3724	0.0000	3.1908	0.0000	1.0836
2008 12%	Backtesting	2	4	6	5	6	1	2	2	2
	Loss 1	0.0087	0.2261	0.4764	0.2379	0.1179	0.0000	0.0633	0.2237	0.1628
	Loss 2	2.0087	4.2261	6.4764	5.2379	6.1179	1.0000	2.0633	2.2237	2.1628
2009 6.96%	Backtesting	2	3	6	5	4	1	1	1	1
	Loss 1	0.1188	0.2263	0.5096	0.3515	0.2568	0.0024	0.1086	0.0216	0.0443
	Loss 2	2.1188	3.2263	6.5096	5.3515	4.2568	1.0024	1.1086	1.0216	1.0443
2009-11.99%	Backtesting	3	3	6	5	5	1	1	1	1
	Loss 1	0.2066	0.3215	0.5654	0.4112	0.3166	0.0299	0.2047	0.0779	0.0647
	Loss 2	3.2066	3.3215	6.5654	5.4112	5.3166	1.0299	1.2047	1.0779	1.0647
2010 6.20%	Backtesting	1	3	5	6	5	0	1	0	1
	Loss 1	0.0161	0.1124	0.4082	0.2268	0.1308	0.0000	0.0260	0.0000	0.0026
	Loss 2	1.0161	3.1124	5.4082	6.2268	5.1308	0.0000	1.0260	0.0000	1.0026
2010 7.55%	Backtesting	1	3	5	6	6	0	1	0	1
	Loss 1	0.0159	0.1258	0.4132	0.2236	0.1239	0.0000	0.0454	0.0000	0.0076
	Loss 2	1.0159	3.1258	5.4132	6.2236	6.1239	0.0000	1.0454	0.0000	1.0076
2011 9.39%	Backtesting	1	3	5	5	6	0	2	0	1
	Loss 1	0.0010	0.1771	0.4492	0.2229	0.1104	0.0000	0.0529	0.0000	0.0025
	Loss 2	1.0010	3.1771	5.4492	5.2229	6.1104	0.0000	2.0529	0.0000	1.0025

Annexure 2: Evaluating Competing VaR Models for Each Selected GOI Bond (*Contd.*)

GOI Bond	Description of Estimate	Normal Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Static Model		Risk Metric with λ						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2011A 11.50%	Backtesting	2	4	6	6	6	0	2	0	2
	Loss 1	0.0048	0.2303	0.4812	0.2388	0.1181	0.0000	0.0709	0.0000	0.0040
	Loss 2	2.0048	4.2303	6.4812	6.2388	6.1181	0.0000	2.0709	0.00000	2.0040
2012 6.85%	Backtesting	1	6	6	6	6	0	3	0	1
	Loss 1	0.0000	0.2866	0.5468	0.2895	0.1691	0.0000	0.1093	0.00000	0.0352
	Loss 2	1.0000	6.2866	6.5468	6.2895	6.1691	0.0000	3.1093	0.00000	1.0352
2012 7.40%	Backtesting	1	6	6	6	6	0	3	0	1
	Loss 1	0.0002	0.3013	0.5510	0.2893	0.1662	0.0000	0.1087	0.0000	0.0324
	Loss 2	1.0002	6.3013	6.5510	6.2893	6.1662	0.0000	3.1087	0.0000	1.0324
2012 - 11.03%	Backtesting	2	6	6	6	7	0	2	0	2
	Loss 1	0.0019	0.3269	0.5453	0.2737	0.1407	0.0000	0.1314	0.0000	0.0162
	Loss 2	2.0019	6.3269	6.5453	6.2737	7.1407	0.0000	2.1314	0.0000	2.0162
2013-7.27%	Backtesting	1	7	6	6	6	0	3	0	2
	Loss 1	0.0050	0.7498	0.8081	0.4783	0.3134	0.0000	0.1498	0.0000	0.0781
	Loss 2	1.0050	7.7498	6.8081	6.4783	6.3134	0.0000	3.1498	0.0000	2.0781
2013 9.81%	Backtesting	1	6	6	6	7	0	3	0	1
	Loss 1	0.0062	0.5711	0.6891	0.3732	0.2195	0.0000	0.1447	0.0000	0.0283
	Loss 2	1.0062	6.5711	6.6891	6.3732	7.2195	0.0000	3.1447	0.0000	1.0283
2014 6.72%	Backtesting	1	6	6	6	6	0	3	0	1
	Loss 1	0.0003	0.3640	0.5951	0.3263	0.1994	0.0000	0.1416	0.0000	0.0431
	Loss 2	1.0003	6.3640	6.5951	6.3263	6.1994	0.0000	3.1416	0.0000	1.0431
2014 7.37%	Backtesting	0	7	6	5	6	0	2	0	2
	Loss 1	0.0000	0.9984	0.9468	0.5777	0.3881	0.0000	0.1481	0.0000	0.1247
	Loss 2	0.0000	7.9984	6.9468	5.5777	6.3881	0.0000	2.1481	0.0000	2.1247
2015 9.85%	Backtesting	2	8	6	6	5	0	3	0	3
	Loss 1	0.0309	1.3841	1.1657	0.7137	0.4847	0.0000	0.2659	0.0000	0.1737
	Loss 2	2.0309	9.3841	7.1657	6.7137	5.4847	0.0000	3.2659	0.0000	3.1737
2015 10.47%	Backtesting	2	7	6	6	5	0	3	0	2
	Loss 1	0.0227	1.1268	0.9980	0.5860	0.3735	0.0000	0.1720	0.0000	0.0839
	Loss 2	2.0227	8.1268	6.9980	6.5860	5.3735	0.0000	3.1720	0.0000	2.0839
2016 10.71%	Backtesting	2	7	6	6	6	0	3	0	2
	Loss 1	0.0387	1.4759	1.2274	0.7774	0.5467	0.0000	0.2590	0.0000	0.0935
	Loss 2	2.0387	8.4759	7.2274	6.7774	6.5467	0.0000	3.2590	0.0000	2.0935

Annexure 2: Evaluating Competing VaR Models for Each Selected GOI Bond (*Concld.*)

GOI Bond	Description of Estimate	*Normal Method*					Historical Simulation		Tail-Index (Hill's Estimator)	
		Static Model		Risk Metric with λ						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2017 7.46%	Backtesting	2	7	7	6	4	0	3	0	2
	Loss 1	0.0404	1.9996	1.8892	1.6647	1.5007	0.0000	0.3675	0.0000	0.1049
	Loss 2	2.0404	8.9996	8.8892	7.6647	5.5007	0.0000	3.3675	0.0000	2.1049
2017 8.07%	Backtesting	2	8	7	6	4	0	2	0	2
	Loss 1	0.0400	1.8312	1.5962	1.2551	1.0537	0.0000	0.3130	0.0000	0.1118
	Loss 2	2.0400	9.8312	8.5962	7.2551	5.0537	0.0000	2.3130	0.0000	2.1118
2018 6.25%	Backtesting	2	6	7	6	4	0	2	0	2
	Loss 1	0.0317	2.1930	2.2607	2.1885	2.0744	0.0000	0.3999	0.0000	0.0818
	Loss 2	2.0317	8.1930	9.2607	8.1885	6.0744	0.0000	2.3999	0.0000	2.0818
2019 10.03%	Backtesting	2	8	7	6	5	0	2	0	2
	Loss 1	0.0626	2.0505	2.1419	2.1280	2.0516	0.0000	0.2459	0.0000	0.0519
	Loss 2	2.0626	10.0505	9.1419	8.1280	7.0516	0.0000	2.2459	0.0000	2.0519
2020 10.70%	Backtesting	2	8	7	6	7	0	3	0	1
	Loss 1	0.0678	2.0705	2.2394	2.3127	2.2777	0.0000	0.2187	0.0000	0.0013
	Loss 2	2.0678	10.0705	9.2394	8.3127	9.2777	0.0000	3.2187	0.0000	1.0013
2022 8.35%	Backtesting	2	7	8	8	7	0	2	0	2
	Loss 1	0.1059	2.4027	3.1476	3.8989	4.1420	0.0000	0.3836	0.0000	0.1019
	Loss 2	2.1059	9.4027	11.1476	11.8989	11.1420	0.0000	2.3836	0.0000	2.1019
2023 6.30%	Backtesting	2	7	7	7	6	0	2	0	1
	Loss 1	0.3238	2.8322	4.0863	5.4880	5.9615	0.0000	0.7997	0.0000	0.2969
	Loss 2	2.3238	9.8322	11.0863	12.4880	11.9615	0.0000	2.7997	0.0000	1.2969
2026 10.18%	Backtesting	4	7	8	5	5	0	4	0	1
	Loss 1	0.5182	2.9683	3.6103	4.9865	5.4668	0.0000	0.7024	0.0000	0.2346
	Loss 2	4.5182	9.9683	11.6103	9.9865	10.4668	0.0000	4.7024	0.0000	1.2346
2032 7.95%	Backtesting	8	9	8	7	3	2	4	2	2
	Loss 1	5.1036	8.3629	6.7324	8.0790	8.8084	0.7251	3.8229	0.69525	0.8605
	Loss 2	13.1036	17.3629	14.7324	15.0790	11.8084	2.7251	7.8229	2.69525	2.8605

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