Value at Risk: Concept and Its Implementation for Indian Banking System

by

Golaka C Nath* and G. P. Samanta#

* *Manager, National Stock Exchange of India Limited (NSEIL), Mumbai (email: golak@nse.co.in).*

Assistant Adviser, Reserve Bank of India, Mumbai (email: gpsamanta@rbi.org.in; rbigps@yahoo.com). The views expressed here are the views of the authors and not necessarily of their respective employers.

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Abstract: Value-at-Risk (VaR) has been widely promoted by the Bank for International Settlement (BIS) as well as central banks of all countries as a way of monitoring and managing market risk and as a basis for setting regulatory minimum capital standards. The revised Basle Accord, implemented in January 1998, makes it mandatory for banks to use VaR as a basis for determining the amount of regulatory capital adequate for covering market risk. For market participants like Banks and Primary Dealers (PDs) in the Indian financial sector, it has become imperative to use VaR methods to calculate the regulatory capital charge required. The RBI has issued guidelines for PDs. We have adopted three categories of VaR methods, viz., Variance-Covariance (Normal) methods including Risk-Metric, Historical Simulation (HS) and Tail-Index Based approach. The Zero-Coupon Yield Curve (ZCYC) compiled by National Stock Exchange (NSE) has been used to price the bonds as well as portfolios. Estimated VaRs are validated by carrying out 'back testing' based on last one year's data. Empirical results show that normal methods, in particularly the Risk-Metric approach, underestimate VaR numbers substantially resulting to too many failures in backtesting. Historical simulation provides more accurate VaR estimates, and indicates capital charge higher than those obtained through normal methods. The tail-index (Hill's estimator) based method also perform quite well though at times it provides too conservative VaR estimates – higher than all other competing VaR models considered here and occasions of VaR violation in backtesting are less than expected number for each bond/portfolio.

Key Words: Market Risk, Value-at-Risk, VaR Models.

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* *Manager, National Stock Exchange of India Limited (NSEIL), Mumbai (email: golak@nse.co.in; gcnath@yahoo.com).*

Assistant Adviser, Reserve Bank of India, Mumbai (email: gpsamanta@rbi.org.in; rbigps@yahoo.com). The views expressed here are the views of the authors and not necessarily of their respective employers. The usual disclaimer applies.

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1. Introduction

Financial institutions are subject to different types of risk, such as, business risk, strategic risk, financial risk and financial risk is one that is caused by movements in financial markets (van den Goorbergh, 1999). The literature distinguishes four major categories of financial risk, viz., credit risk, operational risk, liquidity risk and market risk. Credit risk generally relates to the potential loss due to the default on the part of the counterparty to meet its obligations at designated time. It has three basic components: credit exposure, probability of default and loss in the event of default. Operational risk takes into account the errors that can be made in instructing payments or settling transactions, and includes the risk of fraud and regulatory risks. Liquidity risk is caused by an unexpected large and stressful negative cash flow over a short period. If a firm has highly illiquid assets and suddenly needs some liquidity, it may be compelled to sell some of its assets at a discount. Finally, market risk estimates the loss of an investment portfolio due to the changes in prices of financial assets and liabilities (market conditions).

Monitoring market risk assumes importance to banks and financial institutions, as the values of investment portfolios they hold undergo changes as and when market conditions change. Measuring market risk is important from the viewpoint of devising risk management strategy and for assessing total financial risk (which includes all different types of risks) of an investment portfolio held by a bank or financial institution. There is a need to provide capital charge for this category of risk also so that the banks/institutions remain in business in adverse market conditions. Recognising this point the Bank for International Settlements (BIS) has included market risk as a part of the total risk for which capital has to be provided by a bank.

In recent years, Value at Risk (VaR) has become the standard measure that financial analysts use to quantify the market risk. VaR is commonly defined as the maximum potential fall in value of a portfolio (i.e. loss in portfolio) of financial instruments with a given probability over a certain horizon. In simpler words, it is a number that indicates how much a financial institution can lose with probability, say p, over a given time horizon. The great popularity that this instrument has achieved among financial practitioners is essentially due to its conceptual simplicity: VaR reduces the (market) risk associated with any portfolio to just one number that is the loss associated with a given probability and horizon.

VaR measures can have many applications. It evaluates the performance of risk takers and satisfies the regulatory requirements. VaR has become an indispensable tool for monitoring risk and an integral part of methodologies that allocate capital to various lines of business. Today regulators all over the globe have been forcing institutions to adopt internal models and calculate the required capital charge based on VaR methodologies. In particular, the Basel Committee on Banking Supervision (1996) of the BIS imposes requirements on banks to meet capital requirements based on the VaR estimated through internal model approach. Under this approach, regulators do not provide any specific VaR measurement technique to their supervised banks – the banks are free to use their own model. But to eliminate the possible inertia of supervised banks to underestimate VaR so as to reduce the capital requirements, BIS has prescribed certain minimum standard of VaR estimates and also certain tests, such as backtesting, of VaR models. If VaR model of a bank fails in backtesting, a penalty is imposed resulting to higher capital charge.

Thus, providing accurate estimates of VaR is of crucial importance for all stakeholders. If the underlying risk is not properly estimated, this may lead to a sub-optimal capital allocation with consequences on the profitability or the financial stability of the institutions. A bank would like to pick up a model that would generate as low VaR as possible but pass through the backtesting.

From a statistical point of view, VaR estimation entails the estimation of a quantile of the distribution of returns. Though, there has been voluminous work done on VaR in financial market all over the world, the task of estimating/forecasting VaR still remains challenging. The major difficulty lies in modelling/approximating the return distribution, which generally is not normal (being skewed and/or having fatter tails than normal distribution). Available VaR models can be classified into four broad categories: the historical simulation method, the Monte Carlo simulation method, modelling return distribution (including the variance/covariance method, which assumes normality of the return distribution, and methods under Extreme Value Theory (EVT). All these VaR estimation methods adopt the classical approach: they deal with the statistical distribution of time series of returns.

The main objective of this paper is to discuss issues regarding implementation of VaR models for the portfolios of Govt. of India (GOI) securities held by banks and Primary Dealers (PDs). Reserve Bank of India (RBI), the Central Bank in the country, has issued detailed guidelines on market risk for banks on the basis of the BIS framework though it has not become mandatory for banks to use VaR models[1]. However, RBI has issued detailed guidelines to Primary Dealers (PDs) for mandatory implementation of VaR methods while calculating the capital charge required[2]. We have restricted our analysis to only application of VaR methodologies to GOI bonds and have used the NSE Zero Coupon Yield Curve (ZCYC) parameters to value the bonds as well as the portfolios of bonds so as to construct historical price data for different bonds/portfolios.

The paper has been designed as follows: Section 2 presents a brief review of literature on the subject, Section 3 discusses the theoretical and methodological issues concerning VaR, Section 4 focuses on data and construction of portfolio, Section 5 discusses empirical results and Section 6 concludes.

2. Literature Review

There has been large volume of literature on VaR methodologies as well as on its implementation. The concept received tremendous response from banks all over the world. Banks management can apply the VaR concept to set capital requirements because VaR models allow for an estimate of capital loss due to market risk (see Duffie and Pan, 1997; Jackson, Maude and Perraudin, 1997; Jorion, 1997; Saunders, 1999; Friedmann and Sanddrof-Kohle, 2000; Hartmann-Wendels, et al., 2000; Simons, 2000, among others).

[1] RBI circular BP./21.04.103/2001 dated March 26, 2002.

[2] RBI circular IDMC.PDRS.PDC.3/03.64.00/2000-01 dated December 11, 2000.

The computation of volatility is the most important aspect of any VaR estimation. The volatility estimation should take care of the most stylized facts of any financial asset class – the important ones being fat tailed property, volatility clustering and asymmetry of return distribution. Once these issues are identified in the distribution, then calculating volatility is easy. Today GARCH family models have been increasingly used by researchers to model volatility. An important documentation in this regard has been the J P Morgan's RiskMetrics that applied declining weights to past returns to compute volatility with a decay factor 0.94 which is a variant of IGARCH. Other measures of volatility, which differs from the estimation of return variance, include Garman and Klass (1980), and Gallant and Tauchen (1998), who incorporate daily high and low quotes, and Andersen and Bollerslev (1998) and Andersen, et al. (1999), who use average intraday squared returns to estimate daily volatility.

Several studies such as Danielsson and de Vries (1997), Christoffersen (1998), and Engle and Manganelli (1999) have found significant improvements possible when deviations from the relatively rigid RiskMetrics framework are explored. Choosing an appropriate VaR measure is an important and difficult task, and risk managers have coined the term Model Risk to cover the hazards from working with potentially mis-specified models. Beder (1995), for example, compares simulation-based and parametric models on fixed income and stock option portfolios and finds apparently economically large differences in the VaRs from different models applied to the same portfolio. Hendricks (1996) finds similar results analyzing foreign exchange portfolios. In Indian context, Darbha (2001) made a comparative study of three models – Normal, HS and Extreme Value Theory while studying the portfolio of Gilts held by PDs.

3. Theoretical Issues

As stated earlier, VaR is the maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence and typically calculated for a one-day time horizon with 95% or 99% confidence level. Holding period is one of the most key elements in VaR estimation and the same is chosen on the basis of time that an organization would take to liquidate its position if the need arises. In a very liquid market, 1-day may holding period seem to be justified while in an illiquid market; it may take more than 10 days to liquidate the portfolio. Hence the capital charge would be different for different holding period.

BIS requires that VaR be computed daily by Banks, using a 99th percentile, one-tailed confidence interval with a minimum price shock equivalent to ten trading days (holding period) and the model incorporate a historical observation period of at least one year. The capital charge for a bank that uses a proprietary model will be higher of (i) The previous day's VaR and (ii) an average of the daily VaR of the preceding sixty business days, multiplied by a multiplication factor. The multiplication factor may be 3 and this may go up if the regulators feel that 3 is not sufficient to account for potential weaknesses in the modeling process.

In the case of PDs, RBI prescribes all these above criteria except that (i) minimum holding period would be thirty trading days; (ii) the minimum length of the historical observation period used for calculating VaR should be one year or 250 trading days. For PDs who use a weighting scheme or other methods for the historical observation period, the "effective" observation period must be at least one year (that is, the weighted average time lag of the individual observations cannot be less than 6 months); and (iii) the multiplication factor is presently fixed at 3.3.

The weaknesses may be due to (a) market prices often display patterns (heteroskedastic) that differs from the statistical simplifications used in modeling, (b) past not being always a good approximation of the future (October 1987 crash happened that did not have parallel in historical data), (c) most of the models take ex-post volatility and not ex-ante, (d) VaR estimations normally is based on end-of-day positions and not take into account intra-day risk, (e) models can not adequately capture event risk arising from exceptional market circumstances. Since VaR heavily relies on the availability of historical market price data on the portfolio to understand its effectiveness, it would be appropriate to use the long historical data to see if the stress conditions can be replicated.

While simple VaR models can be implemented for equity and foreign exchange markets as it is not difficult to construct the return series based on actual price data from the secondary markets, it is difficult to use the same concept in fixed income securities. Bonds change their value everyday (assuming *ceteris paribus*) because of time to maturity come down everyday and hence the Present-Value (PV) changes. Hence a 10-year Government of India (GOI) paper today was not a 10-year paper one year back. If an investor would like to find out the VaR numbers for GS CG2011 11.50% with last 3 years historical price data, logically he cannot take the actual trading price data from the secondary market to estimate the VaR numbers. Hence, the logic behind VaR is to consider today's portfolio and find out what would have been its historical values (time series) and then construct the return series and then calculate the VaR numbers. Hence for estimation of VaR on fixed Income securities, there is no other way but to reconstruct the historical data using the time series of yield curves. And for doing the same, either one has to spend resources in establishing a system that will generate the yield curves time series using historical secondary market trades or use NSE ZCYC parameters which have been made available free of cost. NSE has developed the ZCYC using Nelson-Siegel functional form (Nelson and Siegel, 1987) and has created a database from January 1997. Because of parsimony we have used NSE ZCYC parameters to price the bonds and portfolios from 1997.

Estimation of VaR for an individual bond is simple. But when we calculate the VaR of a portfolio, we need to find out the single price series of the portfolio using the historical yield curves. While constructing the portfolio, we have used the weights being the share of the value of a component bonds in the total portfolio value. While constructing the historical price series of the portfolio, it is logical to assume that weights have remained as it is today. This is because VaR envisages to find out the possible maximum loss of a portfolio today in a given time horizon with a level of probability and hence the portfolio needs to be maintained as of today's composition.

3.1. Basic Statistics Related to VaR

The portfolio consists of many securities and in our case we are concerned with only Gilts. The basic price equation of the portfolio can be written as follows:

$$\Pr ice_{portfolio} = w_1 * \Pr_{bond1} + w_2 * \Pr_{bond2} + \ldots\ldots.. + w_n * \Pr_{bondn} \qquad \textbf{..... (1)}$$

and the return on the portfolio is at time defined as

$$R_{pf,t+1} = \sum_{i=1}^{n} w_i * R_{i,t+1} \qquad \text{..... (2)}$$

where the sum is taken over n securities in the portfolio, w_i denotes the proportionate value of the holding of security *i* at the end of day *t*.

And the variance of the portfolio should be written as

$$\sigma^2{}_{PF,t+1} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i * w_j * \sigma_{ij,t+1} = \sum_{i=1}^{n} \sum_{j=2}^{n} w_i * w_j * \sigma_{i,t+1}\sigma_{j,t+1} * \rho_{ij,t+1} \qquad \text{..... (3)}$$

where $\sigma_{ij,t+1}$ is the covariance and $\rho_{ij,t+1}$ is the correlation between security *i* and *j* on day *t+1* and for $\rho_{ij,t+1} = 1$ and we write $\sigma_{ij,t+1} = \sigma^2_{i,t+1}$ for all i.

The VaR of the portfolio is simply

$$VaR^p{}_{PF,t+1} = \sigma_{PF,t+1} * F_p^{-1} \qquad \text{..... (4)}$$

where F_p^{-1} is the p'th quantile of the rescaled portfolio returns.

3.2. Select VaR Methodologies

There are few VaR methodologies that are very simple and easy to implement, to name a few are (a) Normal (parametric using variance and covariance approach) and (b) Historical simulation. Cleverly these simple methods have been extended with application of weights – recent events are given more weight and past is given less. However, different people have used different weighting methodologies. Riskmetrics has used 'exponentially moving average' where the decay factor (λ) has been considered as 0.94 while Boudoukh, et al. (1997) fixed it at 0.98. We will discuss all these issues shortly and calculate the VaR number and see how they are comparable.

There are also complex methods like EVT and Expected Shortfalls that require higher computing skills but not difficult to implement. EVT has two lines of thought – (a) simpler being the block maxima/minima and generalized extreme value in a Pareto optimality framework and (b) the Hill estimator and modeling both sides of the tail separately.

3.2.1. Variance-Covariance (Normal) Method

The Variance-Covariance (Normal) method is the easiest of the VaR methodologies. Since we are considering Gilts for our analysis, it is known that interest rate movement in sovereign bond market is unidirectional at any point of time. If the interest rate changes, it affects the price of all bonds in the similar direction. It will not happen, that a 10-year paper will increase while a 15-year paper will decrease due to an interest rate cut, may be the fall or rise will not be linear. In case of equities, the price of a stock may increase while that of other will fall and there we will surely need correlation coefficient while calculating the volatility of the portfolio. For bonds, the plain standard deviation would be useful to calculate the require VaR. But whether to take static variance of the entire time series or conditional variance is a point for debate. It is argued that variance changes over time horizons and hence we should not rely on unconditional variance for measuring VaR. We will ok at both the options.

The normal method assumes normality in the financial time series. In recent past interest in econometrics and empirical finance has revolved around modeling the temporal variation in financial market volatility. Probability distributions for asset returns often exhibit fatter tails than the standard normal, or Gaussian, distribution. The fat tail phenomenon is known as excess kurtosis. Time series that exhibit a fat tail distribution are often commonly referred to as leptokurtic. In addition, financial time series usually exhibit a characteristic known as volatility clustering, in which large changes tend to follow large changes, and small changes tend to follow small changes. In either case, the changes from one period to the next are typically of unpredictable sign. Large disturbances, positive or negative, become part of the information set used to construct the variance forecast of the next period's disturbance. In this manner, large shocks of either sign are allowed to persist, and can influence the volatility forecasts for several periods. Volatility clustering, or persistence, suggests a time-series model in which successive disturbances are serially correlated.

The volatility-clustering phenomenon can be captured through modelling conditional heteroscadasticity, assuming normality of the conditional distribution of return. A useful class of such time series model includes ARCH/GARCH or some of their further generalisation. This class of models not only handle volatility clustering but also accounts to a great extent the fat tail effect (or excess kurtosis) typically observed in financial data. The popular Risk-Metric model (J.P.Morgan, 1996) is a simplified form of heteroskedasticity. The Risk-Metric approach actually model conditional variance as a weighted average of past variance and past returns, where exponential weighting scheme for past returns is used as follows.

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\, r_t^2 = \lambda^t \sigma_0^2 + (1-\lambda)\sum_{k=0}^{t-1} \lambda^k\, r_{t-k}^2 \qquad \text{..... (5)}$$

where σ_t^2 and r_t denote conditional variance and return at time t, respectively; and the parameter λ, known as decay factor, satisfy $0 < \lambda < 1$.

For daily data, the value of the decay parameter in the RiskMetric approach is generally fixed at λ=0.94 (van den Goorberg and Vlaar, 1999).

3.2.2. Historical Simulation Method

Historical simulation approach provides some advantages over the normal method, as it is not model based, although it is a statistical measure of potential loss. The main benefit is that it can cope with all portfolios that are either linear or non-linear. The method does not assume any specific form of the distribution of price change/return. The method captures the characteristics of the price change distribution of the portfolio, as VaR is estimated on the basis of actual distribution. This is very important, as the HS method would be on the basis of available past data. If the past data does not contain highly volatile periods, then HS method would not be able to capture the same. Hence, HS should be applied when we have very large data points that are sufficiently large to take into account all possible cyclical events. HS method takes a portfolio at a point of time and then revalues the same using the historical price series. Once we calculate the daily returns of the price series, then sorting the same in an ascending order and find out the required data point at desired percentiles. Linear interpolation can be used if the required percentile falls in between 2 data points. The moot question is what length of price series should be used to compute VaR using HS method and what we should do if the price history is not available. It has to be kept in mind that HS method does not allow for time-varying volatility.

Another variant of HS method is a hybrid approach put forward by Boudhoukh, et al. (1997), that takes into account the exponential declining weights as well as HS by extimating the percentiles of the return directly, using declining weights on past data. As described by Boudhoukh et al. (1997, pp. 3), "the approach starts with ordering the returns over the observation period just like the HS approach. While the HS approach attributes equal weights to each observation in building the conditional empirical distribution, the hybrid approach attributes exponentially declining weights to historical returns". The process is simplified as follows:

- Calculate the return series of past price data of the security or the portfolio from *t-1* to *t.*
- To each most recent K returns: *R(t),* R(*t-1*), ……R(*t-K+1*) assign a weight $[(1-\lambda)/(1-\lambda^k)],[(1-\lambda)/(1-\lambda^k)]\lambda,.....[(1-\lambda)/(1-\lambda^k)]\lambda^{k-1}$ respectively. The constant $[(1-\lambda)/(1-\lambda^k)]$ simply ensures that the weights sum to 1.
- Sort the returns in ascending order.
- In order to obtain *p*% VaR of the portfolio, start from the lowest return and keep accumulating the weights until *p*% is reached. Linear interpolation may be used to achieve exactly *p*% of the distribution.
- In many studies lambda (λ) has been used as 0.98.

3.2.3. Extreme Value Theory – Hill's Estimator and VaR Estimation

In financial literature, it is widely believed that high frequency return has fatter tails than can be explained by the normal distribution. The tail-index measures the amount of tail fatness of return distribution and fit within the extreme value theory (EVT). One can therefore, estimate the tail-index and measure VaR based on that. The basic premises of this idea stems from the result that the tails of every fat-tailed distribution converge to the tails of Pareto distribution. The upper tail of such a distribution can be modeled as,

$$\text{Prob}[X > x] \approx C^{\alpha}\,|x|^{-\alpha} \quad (\text{i.e. Prob}[X \le x] \approx 1 - C^{\alpha}\,|x|^{-\alpha});\quad x > C \qquad \textbf{..... (6)}$$

Where, C is a threshold above which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

Similarly, lower tail of a fat-tailed distribution can be modeled as

$$\text{Prob}[X > x] \approx 1 - C^{\alpha}\,|x|^{-\alpha} \quad (\text{i.e. Prob}[X \le x] \approx C^{\alpha}\,|x|^{-\alpha});\quad x < C \qquad \textbf{..... (7)}$$

Where, C is a threshold below which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

In practice, observations in upper tail of the return distribution are generally positive and those in lower tail are negative. Thus, both of equation (6) and equation (7) have importance in VaR measurement. The holder of a short financial position suffers a loss when return is positive and therefore, concentrates on upper-tail of return distribution (i.e. equation 6) while calculating his VaR (Tsay, 2002, pp. 258). Similarly, the holder of a long financial position would model the lower-tail of return distribution (i.e. use equation 7) as a negative return makes him suffer a loss.

From equation (6) and (7), it is clear that the estimation of VaR is crucially dependent on the estimation of tail-index α. There are several methods of estimating tail-index and in the present paper, we consider two approaches, viz. (i) Hill's (1975) estimator and (ii) the estimator under ordinary least square (OLS) framework suggested by van den Goorbergh (1999). We consider here the widely used Hill's estimator, a discussion on which is given below.

Hill's Estimator

For given threshold C in right-tail, Hill (1975) introduced a maximum likelihood estimator of $\gamma = 1/\alpha$ as

$$\hat{\gamma} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{X_i}{C}\right) \qquad \textbf{..... (8)}$$

where X_i's, i=1,2, …..,n are n observations (exceeding C) from the right-tail of the distribution.

In practice, however, C is unknown and needs to be estimated. If sample observations come from Pareto distribution, then C would be estimated by the minimum observed value, the minimum order statistic. However, here we are not modeling complete portion of Pareto distribution. We are only dealing with a fat-tailed distribution that has right tail that is approximated by the tail of a Pareto distribution. As a consequence, one has to select a threshold level, say C, above which the Pareto law holds. In practice, equation (8) is evaluated based on order statistics in the right-tail and thus, the selection of the order statistics truncation number assumes importance. In other words, one needs to select the number of extreme observations n to operationalise equation (8). Mills (1999, pp. 186) discusses a number of available strategies for selecting n and a useful technique for the purpose is due to Phillips, et al. (1996). This approach makes an optimal choice of n that minimises the MSE of the limiting distribution of $\hat{\gamma}$. To implement this strategy, we need estimates of γ for truncation numbers $n_1 = N^{\delta}$ and $n_2=N^{\tau}$, where $0 < \delta < 2/3 < \tau < 1$. Let $\hat{\gamma}_j$ be the estimate of γ for n =n_j, j=1,2. Then the optimal choice for truncation number is n = [λ $T^{2/3}$], where λ is estimated as $\hat{\lambda} =|(\hat{\gamma}_1/\sqrt{2})(T/n_2)(\hat{\gamma}_1 - \hat{\gamma}_2)|^{2/3}$. Phillips et al. (1996) recommended setting δ =0.6 and τ = 0.9 (see Mills, 1999, pp. 186).

Estimating VaR Using Hill's Estimator

Once tail-index α is estimated, the VaR can be estimated as follows (van den Goorbergh and Vlaar, 1999). Let p and q (p < q) be two tail probabilities and x_p and x_q are corresponding quantiles. Then $p \approx C^{\alpha} (x_p)^{-\alpha}$ and $q \approx C^{\alpha} (x_p)^{-\alpha}$ indicating that $x_p \approx x_q (q/p)^{1/\alpha}$. Assuming that the threshold in the left-tail of the return distribution corresponds to the m-th order statistics (in assending order), the estimate of x_p be

$$\hat{x}_p = R_{(m)}\left(\frac{m}{np}\right)^{\hat{\gamma}} \qquad \textbf{..... (9)}$$

where $R_{(m)}$ is the m-th order statistics in the assending order of n observations chosen from tail of the underlying distribution; p is the given confidence level for which VaR is being estimated; $\hat{\gamma}$ is the estimate of γ.

The estimate of VaR (with meanings of notations as defined above) would be

$\hat{V}^{p}_{t+1|t}$ = - W_t $\hat{x}_p$; $\hat{x}_p$ is estimate of quantile of return distribution **….(10)**

or

$\hat{V}^{p}_{t+1|t}$ = W_t [1 – exp($\hat{x}_p$)]; $\hat{x}_p$ is estimate of quantile of log-return distribution **….(11)**

The methodology described above estimates tail-index and VaR for right tail of a distribution. To estimate the parameters for left tail, we simply multiply the observations by –1 and repeat the calculations.

3.3. Estimating Multi-Period VaR from one-period VaR

In practice above methods are used to estimate VaR numbers daily based on one-day holding period returns. However, for computing capital charge, we need the VaR numbers for longer holding period, say 10-days or 30-days. Using the estimates of 1-period VaR, k-period VaR can be estimated by following approximation;

$$\mathrm{VaR(k)} \approx \begin{cases} \left(\sqrt[\alpha]{k}\right)\mathrm{VaR(1)} & \text{if VaR(1) is estimated through tail} - \text{index } \alpha \\ \left(\sqrt{k}\right)\mathrm{VaR(1)} & \text{for other VaR Models} \end{cases} \quad \text{…..(12)}$$

Note from equation (12) that, for the tail-index based VaR model, multi-period VaR VaR(k) depends upon VaR(1), k and the estimated tail index. In the cases of other VaR models, however, VaR(k) can be approximated based on only k and VaR(1). For these relationships one can verify that for given k, higher value of VaR(1) estimated through any VaR model other than tail-index, would indicate higher value of VaR(k). In other words, in this case if VaR(1) for "portfolio 1" is higher than 'portfolio 2" then VaR(k) for "portfolio 1" will also be higher than VaR(k) for "portfolio 2". In the case of tail-index based VaR estimates, however, this does not holds in general. Because, the parameter α also plays an important role here.

3.4. Evaluation of VaR Models - Back Testing

Any method used for VaR estimation need to satisfy the criteria of back testing using the current data set. Suppose we calculate the VaR numbers with probability level 0.01. We can check the accuracy of a VaR model by counting the number of times VaR estimate fails (i.e. actual loss exceeds estimated VaR), say in 100 days. If we want to calculate VaR of a one-day holding period with 99% confidence level, logically, we are allowing 1 failure in 100 days. But if the number is more than 1, then the model is under predicting VaR numbers and if we find less number of failures the model is over predicting. The Basle Committee provides guidelines for imposing penalty leading to higher multiplication factor, when the number of failure is too high. However, no penalty is imposed when the failure occurs with less frequency than the expected number. Thus, selection of VaR model is a very difficult task. A model, which overestimates VaR, may result in reduced number of failure but increase the required capital charge directly. On the other hand if a underestimates VaR numbers, the number failures may be too large which ultimately increases the multiplying factor and hence the required capital charge. Thus an ideal VaR model would be the one, which produces VaR estimates, as minimum as possible and also pass through the backtesting.

The BIS requires that models must incorporate past 250 days data points (one year assuming Saturday/Sundays being non-trading days). In Indian market, RBI has issued guidelines for PDs to use one year and not less than 250 trading days for VaR estimation. Since Saturday is a trading day in bond market, we have taken 290 days (a period of about one year) for our analysis. Accordingly the capital charge is the higher of (i) the previous day's value-at-risk number measured according to the above parameters specified in this section and (ii) the average of the daily value-at-risk measures on each of the preceding sixty business days, multiplied by a multiplication factor prescribed by RBI (3.30 presently for PDs).

To do the back testing, we can think of an indicator variable *I(t)* which is one if return of the day is more than the VaR for the previous day and zero otherwise. Average of the indicator variable should be our VaR percent.

4. Data

We have used the GOI bonds outstanding as on June 23, 2003 for our analysis. As of March 2003, Govt. of India had an outstanding issue size of around Rs.6,739,050[3] million and some more amount has already been borrowed by the Government during April – June 2003. The ownership pattern of GOI securities is given in *Table 1*. As can be seen from *Table 1*, commercial banks hold about 61% per cent of GOI securities in recent years. We have assumed that about 65% of the securities are with the banking system since in recent years banks have been heavily investing in Gilts due to low credit off-take and the same should be about Rs.4,380,000 million.

Table 1: Ownership Pattern of GOI Securities

(in per cent of outstanding Rupees at End-March)

	End-March of the Year			
Category of Holders	**1991**	**1995**	**1999**	**2001**
Reserve Bank of India	24.80	2.51	10.90	9.20
Commercial Banks	55.14	68.78	58.92	60.99
LIC	13.46	17.18	18.22	15.52
EPF Scheme	0.90	0.45	1.37	1.34
Others	5.70	11.08	10.59	12.95
Total	100.00	100.00	100.00	100.00

Source: Derived based on Data in **Handbook of Statistics, 2002-03**, *Reserve Bank of India.*

As of May 2003, the investment-deposit ratio of commercial banks stand about 53.8% meaning that more than 50% of their total assets are in investments and as seen in Table 1, commercial banks also hold a major part of GOI Securities. The capital and reserves of all scheduled commercial banks as on March 31, 2002 stood at Rs.881,860 million. It would be worthwhile to estimate VaR taking the full hypothetical portfolio of the Banking system and see how much capital charge is required.

Our empirical analysis relates to (i) two representative portfolios of GOI bonds, one for banks and another for PDs and (ii) certain selected individual GOI bonds. For applying uniform market conventions in valuation and simplicity, we have not taken T-Bills in our portfolio but it can be included as well. Bonds have been selected from both illiquid as well as liquid baskets. Liquid bonds are those bonds where we observe

[3] RBI Annual Report 2002-03, Table 11.9 (pp. 192)

trading regularly while illiquid bonds are infrequently traded. The liquid bonds identification has been done on the basis of FIMMDA guidelines. FIMMDA identifies liquid and semi liquid Government bonds regularly and puts up the same in its website (http://www.fimmda.org) and we have used the information while choosing/constructing GOI bonds/portfolios for our study. The representative portfolio we consider for entire banking sector holds almost all securities issued by the Govt., each having weight proportional to its outstanding issue size. For the representative portfolio for PDs, we assume that they hold all liquid bonds. Apart from these two portfolios, we consider 31 individual GOI bonds consisting of 15 liquid ones having a high cumulative trading share in total trading in Gilts, 12 semi-liquid and 4 illiquid bonds. These bonds spread across all maturity horizons spreading from 2004 to 2032. Table 2 provides the trading behaviour of selected 31 bonds during last 12 months period (July 2002 to June 2003). These 31 bonds account for about 81% of total trading volume in Gilts (Gsecs and T-Bills put together and about 83% of Gsecs only) during the period. We have used the daily trading data published by NSE in their website to calculate the liquidity distribution. The 15 liquid bonds appear in top 20 in terms of trading value while the semi-liquid ones are in next top 20 in terms of trading value while next 4 are from the rest. The bonds have been picked up in a manner to accommodate all time horizons. We have not considered any T-bills though one may like to include them in the portfolio as well.

Table 2: Liquidity Distribution of Selected Bonds (July 2002 – June 2003)

GOI Bond		No. of trades	Traded value in Rs. Million	Percentage to Total
(A). Liquid Bonds				
2017	8.07	21912	1282453.13	10.76
2012	7.40	21174	1252126.56	10.50
2013	9.81	15214	886534.39	7.44
2015	9.85	13466	766462.50	6.43
2017	7.46	12594	730501.99	6.13
2011A	11.5	10800	690248.67	5.79
2011	9.39	11132	667979.84	5.60
2012	11.03	9146	528589.91	4.43
2022	8.35	5902	387016.18	3.25
2010	7.55	5344	337970.33	2.84
2016	10.71	5051	309167.69	2.59
2009	11.99	4392	298350.43	2.50
2026	10.18	3215	228740.48	1.92
2018	6.25	2905	207368.51	1.74
2013	7.27	3035	201175.19	1.69
(B). Semi-Liquid Bonds				
2004	12.50	1152	128627.58	1.08
2012	6.85	1620	127391.35	1.07
2019	10.03	1430	120540.60	1.01
2014	7.37	1292	94864.07	0.80
2008	12	801	67415.60	0.57
2015	10.47	739	49810.76	0.42
2032	7.95	701	48711.79	0.41
2007	11.90	559	46340.00	0.39
2014	6.72	457	39800.00	0.33

2009	6.96	607	38268.00	0.32
2005	11.19	331	29070.00	0.24
2023	6.30	431	28179.02	0.24
(C). Illiquid Bonds				
2006	11.68	288	20050.00	0.17
2010	6.20	138	13050.00	0.11
2020	10.70	45	3210.00	0.03
2007	11.50	8	1350.00	0.01
Total of 31 Bonds		155881	9631365	80.80*
* The aggregate percentage may not exactly tally with bond-wise percentage totals due to rounding off.				

In order to estimate VaR for any portfolio of GOI bonds (including portfolio of a single bond) we need to construct time series on value/price of the portfolio. For the valuation of bonds, we used a total of 1874 yield curves (NSE ZCYC) from 01-01-1997, constructed by NSE daily basis. However, there was an apparent estimation problem on 23-05-1997 when suddenly the yields have dropped abnormally and next day increased abnormally. To ensure proper use of data, we had looked at the underlying market and did not observe any abnormal trading behaviour. Hence while using the data we considered the data point of 23-05-1997 as an estimation problem and replaced the same with the average value of previous day and next day model prices and calculated returns accordingly. The return series for any portfolio/bond is the continuously compounded return, which is derived as the first difference of daily observations on logarithm of prices. In this process, we get 1874 daily time series observations on return on each of 31 selected GOI bonds and two portfolios of GOI bonds (one for banks and another for PDs).

5. Empirical Results

5.1. Estimates of VaRs and Capital Charge

In this section we report our estimated VaR figures and corresponding capital charges for each bond and portfolio considered in this study. All calculations are restricted to left-tail (one tailed) of return distribution. We first compute 1-day holding period VaR numbers for the last day in our sample as well as the average of 1-day VaRs in last 60 days. All VaR estimates correspond to the probability level 0.01 (equivalently correspond to the confidence level 0.99). For a given security/portfolio, these two VaRs (i.e. 1-day VaR in last day and 60-day average of 1-day VaR) has been adjusted to arrive at VaR numbers corresponding to two alternative holding periods, viz., h=10-days and h=30-days[4]. Required capital charges are calculated as the maximum of last day's VaR and 3.3 times (as prescribed in the RBI circular for PDs) the average of previous sixty-days' VaRs. Relevant results for the representative portfolios of PDs and banks are given in *Table 3*. Similar results for each selected bond are provided in *Annexure 1*. Results for each individual bond/portfolio are reported in four rows. While the first row provides 1-day holding period VaR estimates obtained by

[4] *As per the Basle Committee guideline (1996), capital charge should be derived based on VaR numbers for probability level 0.01 and holding periods 10-days. The VaR for 10-days holding period, however, are calculated based on 1-day VaR numbers computed daily basis. In India, guidelines issued to PDs maintain all attributes for capital charge computation except that VaR should have 30-days holding period (rather than 10-days holding period prescribed in the Basle Committee).*

competing VaR models, the second row reports the average of 1-day VaR in last 60-days in our database. Next two rows report the required capital charges for 10-days and 30-days holding periods.

An important issue need to be mentioned here is that all VaR estimates provided in Table 3 and Annexure 1 are in percentage form, and thus, may actually be termed as the relative VaR (Wong, et al., 2003), which refers to the percentage of a portfolio value which may be lost after h-holding period with a specified probability (i.e. the probability level of VaR). The absolute VaR (i.e. the VaR expressed in Rupees term) can easily be computed by multiplying the portfolio values with the estimated relative VaR. Similarly, the capital charge can also be represented in two alternative forms, viz., relative (i.e. in percentage) or absolute (i.e. in rupees terms). The additional information we require to convert a relative VaR/capital charge in a day to a corresponding absolute term (i.e. rupees term) figures is the value of the portfolio. For example, from Table 3 we see that the capital charge corresponding to 10-days holding period for the portfolio for PDs in the last day in our dataset (i.e. June 23, 2003), obtained by historical simulation using full sample data has been 15.4050 %. Thus, if the value of the portfolio is Rs. 100 at that day, corresponding capital charge for 10-days holding period is Rs. 15.4050.

Table 3: Estimated VaRs and Capital Charges for Two Hypothetical Portfolios

		Variance-Covariance (Normal) Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Simple (homoscadastic)		Risk Metric with λ (conditional heteroscadasctic)						
Portfolio	Description of Estimate*	Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
PDs	Last day's VaR (1)	1.2345	0.9234	0.7991	0.5057	0.3367	1.4706	1.2273	1.5106	1.8441
	60-days' Average VaR (1)	1.2440	0.9648	1.0749	0.9088	0.7363	1.4762	1.2416	1.5279	1.7133
	Cap Charge, H=10-days	12.9816	10.0683	11.2167	9.4834	7.6842	15.4050	12.9572	15.6810	23.9852
	Cap Charge, H=30-day s	22.4847	17.4389	19.4279	16.4257	13.3093	26.6822	22.4426	26.9453	48.1874
Banks	Last day's VaR (1)	1.0798	0.8297	0.7274	0.4597	0.3018	1.2860	1.0403	1.3132	1.5052
	60-days' Average VaR (1)	1.0880	0.8728	0.9780	0.8287	0.6726	1.2888	1.0702	1.3288	1.6092
	Cap Charge, H=10-days	11.3539	9.1084	10.2055	8.6479	7.0190	13.4493	11.1684	13.6537	23.6612
	Cap Charge, H=30-day s	19.6656	15.7762	17.6764	14.9786	12.1572	23.2948	19.3443	23.4748	48.3536

Note: '*' VaR(1) represents VaR for holding period 1-day and H denotes holding period.

The columns in Table 3 and Annexure 1 are self-explanatory. As can be seen therein, we estimated VaRs and capital charges for five alternative schemes under normal method, one for full sample estimate, one for rolling sample estimate, and three for Risk Metric approach corresponding to three alternative decay factors, λ = 0.98, 0.96 and 0.94. Full sample estimates at any day, say t, are derived based on all returns from day 1 to t. In the case of rolling sample estimates, we fix the size/length of the rolling windows at 500 days[5]. The columns with titles 'Full' and 'Rolling' provide estimates corresponding to full sample and rolling sample, respectively. As regards to historical simulation, we provide both 'full sample' and 'rolling sample' estimates. Same is the case for tail-index (Hill's estimator) approach.

[5] We considered the rolling window size as 500 days though one may like to try with other rolling windows like 750, 1000, 1250,1500, etc. This choice, however, is arbitrary, and one may implement a systematic strategy to arrive at an optimal window length for which VaR estimates are most accurate. The task, however, is tedious and, therefore, we omit such a scheme from our agenda in this paper.

5.2. Back Testing for Competing VaR Models

For evaluating performance of competing VaR models, back testing has been carried out with the daily returns for last 290 days (covering about a period of one year as backtesting observations. Both 'full sample' and 'rolling sample' estimates of VaRs are assessed. The backtesting strategy adopted for the case of rolling sample estimates, is as follows; estimate 1-day VaR using returns for days 1 to 500 and compare the same with the return of the 501-th day, estimate 1-day VaR based on returns on days 2 to 501 and compare the same with 502-th day's return, and so on. In the case of full sample estimates, VaRs at any day, say t, are estimated based on returns for the days 1 to t.

As our VaR estimates have probability level 0.01 and the Backtesting trading days cover 290 daily returns, expected number of failures for a good VaR model (i.e. the number of occasions out of 290 days when actual return is worse than VaR) is 3. In *Table 4*, we report the results of Backtesting for two hypothetical portfolios. Detailed bond-wise results of Backtesting are presented in *Annexure 2*.

Table 4: Results of Back Testing for Two Portfolios

	Variance-Covariance (Normal) Method					Historical Simulation		Tail-Index (Hill's Estimator)	
	Simple (homoscadastic)		Risk Metric with λ (conditional heteroscadasctic)						
Portfolio	Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
PDs	2	8	7	8	7	2	3	0	0
Banks	2	8	7	7	7	2	3	2	0

Note: Number in each cell indicates the number of days (out of 290 backtesting days) when actual loss exceeds the VaR (with probability level 0.01). For a good VaR model, this number would be close to 3.

As can be seen from Table 4, for the hypothetical portfolios for PDs and Banks, VaR models under normality for rolling sample as well as the Risk-Metric approach, perform very poorly, as the number of VaR violation is much higher than the expected number 3. In case of full sample homoscadastic normality and also for HS approach, the failure numbers are closer to 3. The Hill's tail-index based VaRs, however, are relatively more conservative (i.e. higher) and in this case VaR violation takes place in at most 2 days (out of 290 days).

The detailed backtesing results for selected GOI bonds (Annexure 2) also reveal the similar findings. Number of failures in the case of Hill's tail-index based VaR model never exceed the theoretical number 3. For HS method, VaR estimates are also quite good, though for a few cases the VaR violation exceeds 3. The performances of normal-based VaR models, generally, are worse. Particularly, for Risk-Metric approach, the number of VaR violation is too high for each bond.

6. Concluding Remarks

This paper has experimented with a number of available VaR models, such as, variance-covariance/normal (including Risk-Metric approach), historical simulation and tail-index based method for estimating VaR for a number of selected GOI bonds and representative portfolios of GOI bonds for banks and PDs. Valuation of each bond has been done based on the Zero-Coupon Yield Curve compiled by NSE. Empirical results are quite interesting. We found that the VaR models under variance-covariance/normal approach, particularly the Risk-Metric approach, severely underestimate VaR numbers, as is reflected by too many failures (compared to the theoretical expected number) in backtesting. Historical simulation approach provides quite reasonable VaR estimates. On the other hand the tail-index (Hill's estimator) based VaR estimates are slightly overestimated and as a result the number of failures in backtesting is less than the theoretical expectation for each individual bond and portfolio.

Annexure 1: Estimated VaRs and Capital Charges for Each Selected GOI Bond

		Normal Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Simple (Homoscadastic)		Risk-Metric with λ (Conditional Heteroscedastic)						
GOI Bond	**Description of Estimate***	**Full**	**Rolling**	**0.98**	**0.96**	**0.94**	**Full**	**Rolling**	**Full**	**Rolling**
2004-12.50%	Last day's VaR (1)	0.4805	0.4352	0.3933	0.3209	0.2769	0.6191	0.5638	0.6692	0.5939
	60-days' Average VaR (1)	0.4829	0.4630	0.4720	0.4402	0.3976	0.6218	0.5945	0.6760	0.6716
	Cap Charge, H=10-days	5.0396	4.8313	4.9253	4.5935	4.1495	6.4886	6.2043	6.2648	7.5880
	Cap Charge, H=30-day s	8.7288	8.3680	8.5308	7.9562	7.1871	11.2387	10.7462	10.2530	13.6590
2005 11.19%	Last day's VaR (1)	0.7018	0.7000	0.6916	0.5578	0.4782	0.9075	0.7583	0.9323	0.8533
	60-days' Average VaR (1)	0.7046	0.7416	0.8354	0.7729	0.6929	0.9084	0.7917	0.9425	0.9667
	Cap Charge, H=10-days	7.3530	7.7389	8.7181	8.0658	7.2311	9.4797	8.2615	9.2884	9.0365
	Cap Charge, H=30-day s	12.7358	13.4042	15.1001	13.9704	12.5247	16.4192	14.3093	15.6553	14.8626
2006-11.68%	Last day's VaR (1)	0.7345	0.7438	0.7260	0.5788	0.4921	0.9100	0.8265	0.9787	0.9300
	60-days' Average VaR (1)	0.7376	0.7969	0.8830	0.8122	0.7241	0.9280	0.8775	0.9825	1.0277
	Cap Charge, H=10-days	7.6973	8.3159	9.2145	8.4754	7.5565	9.6837	9.1567	10.2046	9.7041
	Cap Charge, H=30-day s	13.3321	14.4036	15.9600	14.6798	13.0883	16.7727	15.8598	17.6352	16.0341
2007 11.90%	Last day's VaR (1)	0.7851	0.7725	0.7228	0.5590	0.4642	0.9569	0.9804	0.9964	1.0556
	60-days' Average VaR (1)	0.7891	0.8379	0.8938	0.8115	0.7147	0.9655	1.1418	1.0216	1.2236
	Cap Charge, H=10-days	8.2350	8.7434	9.3268	8.4685	7.4577	10.0754	11.9151	11.3174	13.2033
	Cap Charge, H=30-day s	14.2634	15.1440	16.1546	14.6679	12.9172	17.4510	20.6375	20.1713	23.2780
2008 11.50%	Last day's VaR (1)	1.5098	1.0460	0.8868	0.5772	0.4018	1.7581	1.3866	1.8519	1.63598
	60-days' Average VaR (1)	1.5213	1.0853	1.1828	1.0093	0.8313	1.7593	1.4336	1.8527	1.7799
	Cap Charge, H=10-days	15.8752	11.3260	12.3429	10.5321	8.6753	18.3587	14.9603	19.9154	22.1639
	Cap Charge, H=30-day s	27.4967	19.6173	21.3785	18.2421	15.0261	31.7982	25.9120	34.9874	41.7688
2008 12%	Last day's VaR (1)	1.1617	0.9028	0.7357	0.4949	0.3552	1.2491	0.9901	1.0252	1.0297
	60-days' Average VaR (1)	1.1702	0.9561	0.9676	0.8372	0.7014	1.2537	1.1248	1.0396	1.1302
	Cap Charge, H=10-days	12.2117	9.9769	10.0969	8.7363	7.3197	13.0831	11.7375	10.8435	10.3659
	Cap Charge, H=30-day s	21.1513	17.2805	17.4884	15.1317	12.6780	22.6606	20.3300	18.7765	16.8863
2009 6.96%	Last day's VaR (1)	1.0085	0.8791	0.7182	0.5149	0.3975	1.3016	1.0183	1.2017	1.1733
	60-days' Average VaR (1)	1.0154	0.9494	0.9204	0.8139	0.6985	1.3043	1.2575	1.2238	1.2759
	Cap Charge, H=10-days	10.5958	9.9075	9.6050	8.4938	7.2888	13.6110	13.1222	13.3388	11.9482
	Cap Charge, H=30-day s	18.3524	17.1603	16.6364	14.7117	12.6246	23.5749	22.7284	23.5891	19.6706

Note: '*' VaR(1) represents VaR for holding period 1-day and H denotes holding period.

Annexure 1: Estimated VaRs and Capital Charges for Each Selected GOI Bond *(Contd.)*										
		Normal Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Simple (Homoscadastic)		Risk-Metric with λ (Conditional Heteroscedastic)						
GOI Bond	**Description of Estimate***	**Full**	**Rolling**	**0.98**	**0.96**	**0.94**	**Full**	**Rolling**	**Full**	**Rolling**
2009-11.99%	Last day's VaR (1)	0.9233	0.8236	0.6937	0.5008	0.3898	1.1904	0.9995	1.12458	1.16083
	60-days' Average VaR (1)	0.9294	0.8892	0.8863	0.7850	0.6745	1.2232	1.2350	1.11883	1.19739
	Cap Charge, H=10-days	9.6989	9.2797	9.2491	8.1922	7.0386	12.7651	12.8875	12.17661	11.26291
	Cap Charge, H=30-day s	16.7991	16.0729	16.0200	14.1893	12.1913	22.1098	22.3219	21.51798	18.57515
2010 6.20%	Last day's VaR (1)	1.1350	0.9335	0.7364	0.5132	0.3837	1.3014	1.0717	1.30988	1.18174
	60-days' Average VaR (1)	1.1431	0.9998	0.9550	0.8370	0.7119	1.3079	1.1709	1.31913	1.31137
	Cap Charge, H=10-days	11.9291	10.4330	9.9660	8.7346	7.4293	13.6481	12.2191	13.45170	11.90420
	Cap Charge, H=30-day s	20.6617	18.0706	17.2616	15.1287	12.8680	23.6393	21.1641	23.04426	19.30165
2010 7.55%	Last day's VaR (1)	1.1282	0.9210	0.7307	0.5059	0.3755	1.3086	1.0438	1.29667	1.17738
	60-days' Average VaR (1)	1.1363	0.9842	0.9501	0.8307	0.7046	1.3104	1.1614	1.30931	1.27918
	Cap Charge, H=10-days	11.8578	10.2704	9.9152	8.6689	7.3529	13.6744	12.1203	13.04730	11.56002
	Cap Charge, H=30-day s	20.5383	17.7889	17.1737	15.0150	12.7356	23.6847	20.9929	22.10749	18.69852
2011 9.39%	Last day's VaR (1)	1.2011	0.9314	0.7491	0.5070	0.3666	1.2622	1.0148	1.37506	1.32403
	60-days' Average VaR (1)	1.2099	0.9866	0.9830	0.8525	0.7167	1.2703	1.1439	1.38566	1.26197
	Cap Charge, H=10-days	12.6262	10.2960	10.2577	8.8965	7.4787	13.2566	11.9372	14.05360	10.81599
	Cap Charge, H=30-day s	21.8692	17.8332	17.7669	15.4092	12.9535	22.9611	20.6759	24.01261	17.07628
2011A 11.50%	Last day's VaR (1)	1.1829	0.9122	0.7431	0.4993	0.3578	1.2604	1.0048	1.37483	1.28499
	60-days' Average VaR (1)	1.1916	0.9650	0.9779	0.8457	0.7083	1.2632	1.1466	1.39295	1.28714
	Cap Charge, H=10-days	12.4349	10.0702	10.2045	8.8256	7.3918	13.1823	11.9653	14.59451	11.80419
	Cap Charge, H=30-day s	21.5378	17.4421	17.6746	15.2864	12.8030	22.8324	20.7244	25.32687	19.23314
2012 6.85%	Last day's VaR (1)	1.3908	1.0162	0.8216	0.5534	0.3990	1.4314	1.1483	1.58557	1.43951
	60-days' Average VaR (1)	1.4012	1.0668	1.0809	0.9354	0.7852	1.4325	1.2144	1.59764	1.48996
	Cap Charge, H=10-days	14.6217	11.1327	11.2796	9.7616	8.1942	14.9488	12.6727	15.90033	14.68881
	Cap Charge, H=30-day s	25.3256	19.2824	19.5368	16.9076	14.1928	25.8921	21.9498	26.92439	24.76522
2012 7.40%	Last day's VaR (1)	1.3733	1.0050	0.8153	0.5480	0.3940	1.4136	1.1407	1.56565	1.39213
	60-days' Average VaR (1)	1.3835	1.0551	1.0735	0.9283	0.7784	1.4162	1.2168	1.58020	1.48224
	Cap Charge, H=10-days	14.4374	11.0107	11.2025	9.6874	8.1230	14.7793	12.6982	15.83250	14.95836
	Cap Charge, H=30-day s	25.0064	19.0711	19.4032	16.7790	14.0695	25.5985	21.9939	26.89545	25.49986

Note: '*' VaR(1) represents VaR for holding period 1-day and H denotes holding period.

Annexure 1: Estimated VaRs and Capital Charges for Each Selected GOI Bond *(Contd.)*										
		Normal Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Simple (Homoscadastic)		Risk-Metric with λ (Conditional Heteroscedastic)						
GOI Bond	Description of Estimate*	Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2012-11.03%	Last day's VaR (1)	1.2594	0.9401	0.7728	0.5150	0.3658	1.3184	1.0687	1.46683	1.34810
	60-days' Average VaR (1)	1.2687	0.9894	1.0204	0.8798	0.7344	1.3280	1.1511	1.48697	1.41105
	Cap Charge, H=10-days	13.2401	10.3253	10.6487	9.1816	7.6637	13.8580	12.0128	15.53409	14.70652
	Cap Charge, H=30-day s	22.9324	17.8840	18.4440	15.9029	13.2739	24.0028	20.8069	26.92002	25.46586
2013-7.27%	Last day's VaR (1)	1.5390	1.0680	0.8893	0.5928	0.4244	1.6707	1.3337	1.86668	1.70292
	60-days' Average VaR (1)	1.5506	1.1107	1.1758	1.0123	0.8445	1.6781	1.3655	1.81013	1.56160
	Cap Charge, H=10-days	16.1809	11.5910	12.2699	10.5634	8.8129	17.5119	14.2497	18.57984	15.28507
	Cap Charge, H=30-day s	28.0261	20.0763	21.2520	18.2963	15.2645	30.3314	24.6811	31.93050	25.73298
2013 9.81%	Last day's VaR (1)	1.3878	0.9934	0.8273	0.5480	0.3878	1.4767	1.2067	1.63045	1.40350
	60-days' Average VaR (1)	1.3983	1.0384	1.0954	0.9418	0.7838	1.4778	1.2848	1.66865	1.45674
	Cap Charge, H=10-days	14.5918	10.8357	11.4314	9.8286	8.1789	15.4213	13.4075	17.54048	14.33306
	Cap Charge, H=30-day s	25.2738	18.7680	19.7998	17.0236	14.1663	26.7105	23.2225	30.48686	24.21780
2014 6.72%	Last day's VaR (1)	1.4332	1.0321	0.8396	0.5647	0.4070	1.4765	1.1930	1.65868	1.45711
	60-days' Average VaR (1)	1.4439	1.0808	1.1054	0.9559	0.8018	1.4920	1.2343	1.67298	1.49680
	Cap Charge, H=10-days	15.0679	11.2789	11.5351	9.9750	8.3670	15.5700	12.8806	17.50132	14.69251
	Cap Charge, H=30-day s	26.0984	19.5356	19.9794	17.2771	14.4921	26.9681	22.3098	30.34875	24.75312
2014 7.37%	Last day's VaR (1)	1.0960	2.2546	0.9194	0.6104	0.4362	1.8353	1.4095	1.92464	1.73926
	60-days' Average VaR (1)	1.1362	2.2722	1.2178	1.0462	0.8705	1.8383	1.4277	1.95134	1.67170
	Cap Charge, H=10-days	11.8564	23.7121	12.7081	10.9173	9.0841	19.1838	14.8983	21.66495	17.37143
	Cap Charge, H=30-day s	20.5359	41.0705	22.0111	18.9094	15.7341	33.2273	25.8046	38.65079	30.11794
2015 9.85%	Last day's VaR (1)	1.6161	1.1038	0.9322	0.6080	0.4263	1.9010	1.4886	1.97999	1.73134
	60-days' Average VaR (1)	1.6284	1.1414	1.2431	1.0603	0.8734	1.9053	1.5156	1.97608	1.71742
	Cap Charge, H=10-days	16.9936	11.9106	12.9720	11.0647	9.1143	19.8832	15.8163	20.73941	18.38116
	Cap Charge, H=30-day s	29.4337	20.6297	22.4682	19.1646	15.7865	34.4387	27.3947	36.02016	32.23363
2015 10.47%	Last day's VaR (1)	1.5256	1.0540	0.8921	0.5831	0.4082	1.7639	1.3883	1.86069	1.77378
	60-days' Average VaR (1)	1.5372	1.0934	1.1880	1.0152	0.8381	1.7679	1.4357	1.87827	1.80238
	Cap Charge, H=10-days	16.0414	11.4106	12.3979	10.5942	8.7456	18.4487	14.9827	20.71056	21.62626
	Cap Charge, H=30-day s	27.7845	19.7638	21.4738	18.3498	15.1479	31.9540	25.9507	36.82918	40.08962

Note: '*' VaR(1) represents VaR for holding period 1-day and H denotes holding period.

Annexure 1: Estimated VaRs and Capital Charges for Each Selected GOI Bond *(Contd.)*

		Normal Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Simple (Homoscadastic)		Risk-Metric with λ (Conditional Heteroscedastic)						
GOI Bond	Description of Estimate*	Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2016 10.71%	Last day's VaR (1)	1.6206	1.1098	0.9362	0.6076	0.4238	1.9180	1.5120	1.95581	1.76379
	60-days' Average VaR (1)	1.6330	1.1470	1.2507	1.0646	0.8744	1.9260	1.5507	1.97107	1.69514
	Cap Charge, H=10-days	17.0417	11.9694	13.0518	11.1099	9.1244	20.0985	16.1821	20.14285	17.63634
	Cap Charge, H=30-day s	29.5170	20.7316	22.6064	19.2429	15.8039	34.8116	28.0282	34.54158	30.52388
2017 7.46%	Last day's VaR (1)	1.9371	1.3094	1.0697	0.6997	0.5012	2.4963	1.7607	2.41831	1.98706
	60-days' Average VaR (1)	1.9520	1.3434	1.4272	1.2124	0.9952	2.4995	1.7620	2.43472	2.02590
	Cap Charge, H=10-days	20.3705	14.0189	14.8940	12.6525	10.3858	26.0834	18.3874	25.25757	21.56280
	Cap Charge, H=30-day s	35.2828	24.2814	25.7971	21.9147	17.9888	45.1778	31.8479	43.62410	37.71066
2017 8.07%	Last day's VaR (1)	1.8341	1.2376	1.0255	0.6699	0.4760	2.2716	1.6790	2.22722	1.75649
	60-days' Average VaR (1)	1.8481	1.2723	1.3684	1.1641	0.9567	2.2837	1.7044	2.25350	1.84146
	Cap Charge, H=10-days	19.2862	13.2776	14.2797	12.1485	9.9841	23.8318	17.7863	22.27434	18.51916
	Cap Charge, H=30-day s	33.4047	22.9974	24.7331	21.0418	17.2930	41.2778	30.8067	37.59434	31.52357
2018 6.25%	Last day's VaR (1)	2.0853	1.4057	1.1294	0.7439	0.5409	2.7461	1.8666	2.60423	2.10819
	60-days' Average VaR (1)	2.1013	1.4389	1.5041	1.2779	1.0505	2.7600	1.8753	2.63395	2.13650
	Cap Charge, H=10-days	21.9281	15.0158	15.6963	13.3357	10.9629	28.8018	19.5695	27.41268	21.72406
	Cap Charge, H=30-day s	37.9806	26.0082	27.1868	23.0982	18.9882	49.8861	33.8954	47.41984	37.16716
2019 10.03%	Last day's VaR (1)	1.9087	1.3409	1.0760	0.7012	0.5048	2.4961	1.8509	2.39103	2.03259
	60-days' Average VaR (1)	1.9234	1.3746	1.4378	1.2153	0.9914	2.5044	1.8601	2.42206	1.98226
	Cap Charge, H=10-days	20.0715	14.3452	15.0044	12.6821	10.3454	26.1347	19.4109	24.84974	19.61311
	Cap Charge, H=30-day s	34.7649	24.8466	25.9884	21.9660	17.9187	45.2665	33.6206	42.69381	33.12515
2020 10.70%	Last day's VaR (1)	1.9196	1.3666	1.0899	0.7133	0.5186	2.4958	1.8752	2.45343	1.91994
	60-days' Average VaR (1)	1.9343	1.4001	1.4544	1.2286	1.0022	2.5128	1.8869	2.46666	1.99694
	Cap Charge, H=10-days	20.1854	14.6103	15.1779	12.8211	10.4581	26.2228	19.6906	26.06636	19.47205
	Cap Charge, H=30-day s	34.9621	25.3057	26.2889	22.2068	18.1140	45.4192	34.1052	45.42003	32.66079
2022 8.35%	Last day's VaR (1)	2.1897	1.6404	1.2642	0.8708	0.6865	2.8581	2.1118	2.82782	2.43702
	60-days' Average VaR (1)	2.2061	1.6706	1.6570	1.4058	1.1615	2.8841	2.1118	2.85296	2.55708
	Cap Charge, H=10-days	23.0217	17.4340	17.2912	14.6699	12.1211	30.0973	22.0379	29.48916	28.80464
	Cap Charge, H=30-day s	39.8747	30.1965	29.9492	25.4089	20.9944	52.1301	38.1708	50.84461	51.75369

Note: '*' VaR(1) represents VaR for holding period 1-day and H denotes holding period.

Annexure 1: Estimated VaRs and Capital Charges for Each Selected GOI Bond *(Concld.)*											
		Normal Method					**Historical Simulation**		**Tail-Index (Hill's Estimator)**		
		Simple (Homoscadastic)		**Risk-Metric with λ (Conditional Heteroscedastic)**						
GOI Bond	**Description of Estimate***	Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2023 6.30%	Last day's VaR (1)	2.4447	1.8962	1.4380	1.0347	0.8588	3.2206	2.3300	3.20569	3.09765
	60-days' Average VaR (1)	2.4624	1.9238	1.8516	1.5807	1.3236	3.2751	2.3300	3.23842	2.92292
	Cap Charge, H=10-days	25.6968	20.0763	19.3222	16.4952	13.8124	34.1773	24.3152	33.53046	31.03540
	Cap Charge, H=30-day s	44.5081	34.7732	33.4670	28.5705	23.9238	59.1969	42.1152	57.86030	54.41294
2026 10.18%	Last day's VaR (1)	2.2651	1.8815	1.5211	1.1600	1.0111	2.9036	2.4249	2.98585	2.89859
	60-days' Average VaR (1)	2.2802	1.9018	1.9048	1.6485	1.4065	2.9191	2.4249	3.01020	2.74671
	Cap Charge, H=10-days	23.7951	19.8464	19.8772	17.2028	14.6771	30.4623	25.3047	31.14444	27.26564
	Cap Charge, H=30-day s	41.2144	34.3749	34.4283	29.7961	25.4215	52.7622	43.8290	53.72337	46.24050
2032 7.95%	Last day's VaR (1)	2.5788	2.5197	2.3594	2.0570	1.9490	3.4786	2.8300	3.48599	3.43732
	60-days' Average VaR (1)	2.5880	2.5067	2.7118	2.4465	2.1937	3.5009	2.8300	3.53876	3.63368
	Cap Charge, H=10-days	27.0070	26.1588	28.2995	25.5304	22.8921	36.5333	29.5330	37.95843	36.42364
	Cap Charge, H=30-day s	46.7775	45.3084	49.0163	44.2199	39.6503	63.2776	51.1527	66.61620	62.01635

Note: '*' VaR(1) represents VaR for holding period 1-day and H denotes holding period.

Annexure 2: Results of Backtesting for Selected GOI Bonds

GOI Bond	Normal Method: Simple (Homoscadastic)		Normal Method: Risk-Metric with λ (Conditional Heteroscedastic)			Historical Simulation		Tail_Index Hill's Estimate)	
	Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
2004-12.50%	3	3	6	5	5	1	2	1	1
2005 11.19%	4	4	6	7	6	2	3	2	2
2006-11.68%	4	3	6	6	6	2	2	2	2
2007 11.90%	4	4	7	8	6	2	2	2	2
2008 11.50%	2	7	6	6	6	0	3	0	1
2008 12%	2	4	6	5	6	1	2	2	2
2009 6.96%	2	3	6	5	4	1	1	1	1
2009-11.99%	3	3	6	5	5	1	1	1	1
2010 6.20%	1	3	5	6	5	0	1	0	1
2010 7.55%	1	3	5	6	6	0	1	0	1
2011 9.39%	1	3	5	5	6	0	2	0	1
2011A 11.50%	2	4	6	6	6	0	2	0	2
2012 6.85%	1	6	6	6	6	0	3	0	1
2012 7.40%	1	6	6	6	6	0	3	0	1
2012 - 11.03%	2	6	6	6	7	0	2	0	2
2013-7.27%	1	6	6	6	7	0	3	0	2
2013 9.81%	1	6	6	6	7	0	3	0	1
2014 6.72%	1	6	6	6	6	0	3	0	1
2014 7.37%	0	7	6	5	6	0	2	0	2
2015 9.85%	2	8	6	6	5	0	3	0	3
2015 10.47%	2	7	6	6	5	0	3	0	2
2016 10.71%	2	7	6	6	6	0	3	0	2
2017 7.46%	2	7	7	6	4	0	3	0	2
2017 8.07%	2	8	7	6	4	0	2	0	2
2018 6.25%	2	6	7	6	4	0	2	0	2
2019 10.03%	2	8	7	6	5	0	2	0	2
2020 10.70%	2	8	7	6	7	0	3	0	1
2022 8.35%	2	7	8	8	7	0	2	0	2
2023 6.30%	2	7	7	7	6	0	2	0	1
2026 10.18%	4	7	8	5	5	0	4	0	1
2032 7.95%	8	9	8	7	3	2	4	2	2

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Author Profiles:

1. Shri Golaka C Nath earned his MA (Economics) from University of Calcutta and MBA (Finance) from Jadavpur University, Kolkata and currently continuing his Ph.D. in Financial Markets at Goa University. He is working as Manager in National Stock Exchange of India Ltd. Mumbai. His area of interest is Financial Markets and Econometrics. He has several publications to his credit.
2. Dr. G P Samanta earned his M.Stat (Master of Statistics) form Indian Statistical Institute, Calcutta and Ph.D in Economics from University of Mumbai. He is currently working as Assistant Advisor in Reserve Bank of India, Mumbai. His areas of research interest include Econometrics and Applied Statistics, Monetary Policy, Modelling and Forecasting, Applied Economics and Finance. He has several publications to his credit.