Yield Curves and Other Stories
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Introduction:

The yield curve, which plots the yield of Gilts against their maturity, is one of the most closely watched financial indicators. Theoretically speaking a yield curve is the graphic or numeric presentation of bond equivalent yields to maturity on debt that is identical in every aspect except time to maturity. In developing a yield curve, default risk and liquidity, for example, are the same for every security whose yield is included in the yield curve. Thus yields on Gilts are normally used to plot yield curves. The relationship between yields and time to maturity is often referred to as the term structure of interest rates. Many market observers carefully track the yield curve's shape, which is typically upward sloping and somewhat convex. At times the curve becomes flat or slopes downward (inverted), configurations that many business economists, financial analysts, and other practitioners regard as harbingers of recession. In most of the days of 2000 and 2001, we have witnessed the inverted yield curve in India. But this inversion only remained for the very shorter end where the mismatch of short term asset an liabilities played a very important role. That could also possibly explain why on many occasions one-day money did cost more than 14-days or 1-month term-money.

Advantages: In fact, forecasting with the yield curve does have a number of advantages. Financial market participants truly value accurate forecasts, since they can mean the difference between a large profit and a large loss. As deregulation of interest rate in India has paved the way for the market to determine the true cost of money, by some measures, the yield curve should be an even better predictor now than it has been in the past. Widespread use of the yield curve makes assessing its accuracy a worthwhile exercise for all. But policymakers, too, need an accurate and timely predictor of future economic growth and indicator of monetary policy directions. The central bank of the country should need it more as the yield curve will provide him the indicative rates at which the Government securities are going to be issued in auction.

The ready availability of term-structure data ensures a timely prediction, but accuracy is another question. Central bankers have an added incentive to understand the yield curve, since the reference rates like Bank Rate and Discount rate are themselves interest rates. And most of the time policies are undertaken using interest rates as an indirect tool by the central bank. Aside from the knowledge gained about the curve itself, there are several reasons as why understanding the workings and utility of a true and accurate yield curve is important. Simple predictions may serve as a check on more complex models, perhaps highlighting when assumptions or relationships need rethinking. There has been considerable debate in the market place as to whether spot curve (zero coupon yield curve) should be more increasingly used by market participants for enacting trading strategies, valuation, etc. Money and Bond market traders have been increasingly reposing their faith on the YTM based curves to decide on their trading strategies. Agreement between predictions increases confidence in the results, while disagreement signals the need for a second look. A simple, popular indicator also provides some insight into market sentiment. Of course, it's always a good idea to check whether the expensive

and complicated forecasts actually do perform better. For a trader, it makes economics sense to use a YTM based curve or a zero coupon curve if only it correctly reveals the market term structure otherwise there will be mis-pricing.

As explained by the expectations hypothesis of the term structure of interest rates, the typical yield curve gradually increases relative to maturity. That is, in normal economic conditions short-term rates are somewhat lower than longer-term rates because a person needs to be compensated for loss of liquidity. In a recession the entire yield curve shifts downward as interest rates generally fall and rotates indicating that short-term rates have fallen to much lower levels than long-term rates. In an economic expansion accompanied by inflation, interest rates tend to rise and yield curves shift upward and rotate indicating that short-term rates have increased more than long-term rates. If low interest rates are associated with recessions, then an inverted term structure—implying that upcoming rates will be lower—predicts a recession. If policymakers act to reduce short-term interest rates in recessions, market participants who expect a recession would also expect low rates.

The yield curve picks up the financial market's estimate of future policy. Another possibility is that current monetary policy may shift both the yield curve and future output. For example, tight monetary policy might raise short-term interest rates, flattening the yield curve and leading to slower future growth. Conversely, easy policy could reduce short-term interest rates, steepen the yield curve, and stimulate future growth. The risk premium provides another reason why the yield curve may be a useful predictor: The premium itself holds information. As a simple example, consider that recessions may make people uncertain about future income and employment, or even about future interest rates. The risk premium on a longer-term bond reflects this. In conjunction with changes working through the expectations hypothesis, the yield curve may take some very strange twists indeed, becoming inverted, humped, or even u-shaped. These explanations provide an additional motivation for investigating yield curve predictions.

Predictive Power: They also hint at the many important issues that transcend the yield curve's predictive power. It matters, for instance, if the curve reacts to future policy, to movements in output, or to some combination of the two. But these considerations fall by the wayside if the yield curve is not an accurate predictor of future economic activity. . In fact, it sometimes helps to draw a distinction between the yield curve and the term structure. The yield curve is the relation between the yield on Gilts and their maturity. The term structure is a particular yield curve that for zero-coupon Gilts. The term structure is theoretically more interesting. It answers the question, "How much would I pay for one rupee delivered 10 years from today?" The problem is that a zero-coupon Govt. security rarely matures in exactly 10 years. What we actually observe in the market are prices (and thus yields) on existing Govt. securities. These may not mature in precisely 10 years (or whatever maturity you choose), and they often have coupon payments. That is, a 10-year Govt. paper pays interest semiannually at a specified coupon rate, so its yield is not the yield called for in the term structure. Finding the desired interest rate almost always involves estimation of some kind.

Practical issues:

Indian bond market has undergone dramatic changes over last 3 years or so and the trading volume in the market has increased many folds. However, the trading has concentrated on Gilts only. Market participants use their own estimation of term structure while trading in the market and the market is by and large relies on YTM for trading strategies. However, some advocate use of zero coupon yield curves to design trading strategies and valuation as YTM concept does not answer some of the issues which ZCYC does. We are not going into the details of the same. The zero coupon yield curve is a theoretical plot of term structure using some established models like Nelson Siegel functional form, Splines, (cubic or B-splines) etc. There are two things you use yield curve modeling for: When pricing new issues and when looking at the relative value of existing bonds. Models that fit the curve (N&S, b-splines and many more) cannot be used for pricing but for relative value. All swaps systems use bootstrapping for calculating zero coupon curves. When it comes to estimating (fitting) curves I think N&S or any other model works fine as long as you know your model (and how it behaves in "extreme" situations).

Let us look for some practical issues involving the subject. Recently Reserve Bank of India issued a 10-year paper on April 10, 2002. As per the RBI stipulations, auction papers were submitted by market participants by 2.30PM when the market was trading. This implies that market participants have already discounted all possible and available information while putting in their bids for the auction. And there is no reason to believe that traders will price otherwise for trading in the secondary market while buying and selling the existing papers. Auction prices will reflect the true yield out of a process where relatively large number of buyers and sellers participate unlike in secondary market where you have more negotiated deals taking place. Hence, auction days will be very appropriate to test the practical usage of an zero coupon yield curve. The 10-year paper was issued at 6.85% coupon. If we take the spot rates as estimated by ZCYC released for the same day and use it for valuation of the streams of cashflows, it would look as follows:

GOI 6.85% 2012 Paper

Date	Coupon / Final Payment	Spot rate as given by ZCYC	PV	PV with Continuous comp.	Half Years
4-Oct-02	3.425	5.667834	3.331879	3.32930077	1
4-Apr-03	3.425	5.778173	3.237908	3.23270659	2
4-Oct-03	3.425	5.887564	3.143353	3.13549801	3
4-Apr-04	3.425	5.995152	3.048517	3.03799707	4
4-Oct-04	3.425	6.100283	2.953719	2.94054341	5
4-Apr-05	3.425	6.202468	2.859280	2.84347656	6
4-Oct-05	3.425	6.301350	2.765507	2.74712307	7
4-Apr-06	3.425	6.396686	2.672687	2.65178773	8
4-Oct-06	3.425	6.488319	2.581081	2.55774780	9
4-Apr-07	3.425	6.576166	2.490922	2.46524985	10
4-Oct-07	3.425	6.660201	2.402411	2.37450834	11
4-Apr-08	3.425	6.740444	2.315723	2.28570568	12
4-Oct-08	3.425	6.816951	2.230999	2.19899328	13
4-Apr-09	3.425	6.889804	2.148358	2.11449322	14
4-Oct-09	3.425	6.959108	2.067889	2.03230046	15
4-Apr-10	3.425	7.024981	1.989662	1.95248520	16
4-Oct-10	3.425	7.087552	1.913723	1.87509545	17
4-Apr-11	3.425	7.146958	1.840102	1.80015951	18
4-Oct-11	3.425	7.203338	1.768812	1.72768843	19
4-Apr-12	103.425	7.256835	51.33053	50.0570445	20
			99.09307	97.3599049	
Now paid			100	100	
Diff			0.906933	2.64009507	

To make things little more lucid for the readers let us take the most recent auction on May 2, 2002 for a 10 year paper with a cut-off coupon of 7.40% and a weighted average yield of 7.39%. The table below will take through the calculations:

GOI 7.40% 2012 Paper:

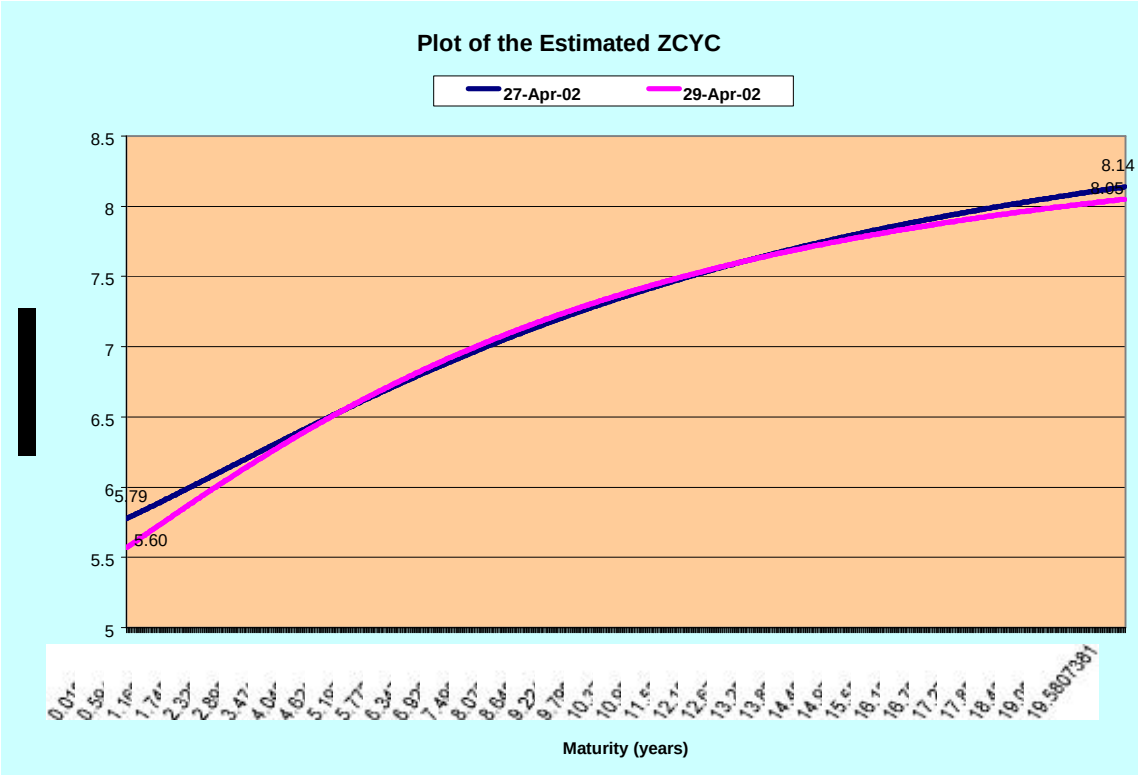
Date	Coupon Payment	Spot rate as given by ZCYC	PV	PV with Continuous comp.	Half Years
2-Nov-02	3.7	5.608888	3.600406	3.597677	1
2-May-03	3.7	5.818378	3.496557	3.490863	2
2-Nov-03	3.7	6.007401	3.389982	3.381170	3
2-May-04	3.7	6.178034	3.281953	3.269942	4
2-Nov-04	3.7	6.332166	3.173511	3.158283	5
2-May-05	3.7	6.471500	3.065502	3.047092	6
2-Nov-05	3.7	6.597573	2.958599	2.937085	7
2-May-06	3.7	6.711768	2.853333	2.828827	8
2-Nov-06	3.7	6.815323	2.750114	2.722752	9
2-May-07	3.7	6.909349	2.649252	2.619191	10
2-Nov-07	3.7	6.994837	2.550975	2.518382	11
2-May-08	3.7	7.072675	2.455443	2.420496	12
2-Nov-08	3.7	7.143654	2.362761	2.325640	13
2-May-09	3.7	7.208479	2.272991	2.233878	14
2-Nov-09	3.7	7.267781	2.186160	2.145235	15
2-May-10	3.7	7.322119	2.102267	2.059706	16
2-Nov-10	3.7	7.371994	2.021287	1.977263	17
2-May-11	3.7	7.417851	1.943182	1.897859	18
2-Nov-11	3.7	7.460087	1.867899	1.821435	19
2-May-12	103.7	7.499056	50.319027	48.989034	20
			101.3012	99.44181	
Amount Paid			100	100	
Diff			-1.301204	0.558188	

As evident from the above tables, the estimation of the term structure is very very different from the reality and it does not reflect the market expectations / condition even though the zero coupon yield curve has been developed using the market prices from the trades taking place on that day. As long as the numbers are not meaningful for the traders, he will find very little incentive to use the same. Hence, even if we develop a zero coupon yield curve using the best possible models and functional form and best possible software, it may be of less use for the market participants as long as the same is not subjected to stress testing conditions.

Another look at the performance of the zero coupon yield curve will also help to understand the situation. Monetary and Credit policy of RBI was issued on April 29, 2002. There was a general correction in prices of Gilts across maturities and yields have gone up as the prices had fallen. The table below gives the trend of the yield for various securities traded on 27th and 29th April 2002.

		29-Apr					27-Apr		
Sec ctype	Security	Issue	Last Traded	Weighted		Security	Issue	Last Traded	Weighted
		Name	Price/ Rate	YTM	Diff. in YTM		Name	Price/Rate	YTM
GS	CG2004	12.50%	111.1050	6.1823	-0.008	CG2004	12.50%	111.1275	6.1906
GS	CG2005	10.20%	110.9700	6.3500	-0.020	CG2005	10.20%	110.9500	6.3700
GS	CG2005	13.75%	120.3160	6.2490		CG2005	11.19%	114.1800	6.3330
GS	CG2005	9.90%	109.4900	6.3490	-0.013	CG2005	9.90%	109.4800	6.3624
GS	CG2006	11.68%	117.9500	6.4480	0.054	CG2006	11.68%	118.1800	6.3940
GS	CG2008	11.40%	122.8000	6.8229	0.019	CG2008	11.40%	123.3800	6.8039
GS	CG2009	6.65%	98.4000	6.9008	0.032	CG2009	6.65%	98.8000	6.8680
GS	CG2010	11.30%	125.1500	7.1970	0.013	CG2010	11.30%	125.3000	7.1840
GS	CG2010	8.75%	107.8000	7.4990		CG2011	10.95%	123.1600	7.4094
GS	CG2011	9.39%	113.9300	7.2381	0.003	CG2011	9.39%	114.2300	7.2351
GS	CG2011A	11.50%	128.2100	7.2782	0.036	CG2011A	11.50%	129.0800	7.2416
GS	CG2012	11.03%	126.2300	7.2981	0.024	CG2012	11.03%	126.8500	7.2738
GS	CG2012	9.40%	114.9500	7.2709	0.034	CG2012	9.40%	115.5500	7.2365
GS	CG2013	9.81%	118.2000	7.3361	0.026	CG2013	9.81%	118.8900	7.3097
GS	CG2014	11.83%	131.7500	7.8153	0.016	CG2014	11.83%	132.0500	7.7987
GS	CG2015	10.47%	122.6200	7.6570	0.051	CG2015	10.47%	123.1000	7.6060
GS	CG2015	9.85%	119.8200	7.4498	0.029	CG2015	9.85%	120.5900	7.4208
GS	CG2016	10.71%	125.8000	7.6157	0.029	CG2016	10.71%	126.7000	7.5866
GS	CG2017	7.49%	100.2000	7.4165	0.014	CG2017	7.49%	101.0000	7.4025
GS	CG2017	8.07%	105.1000	7.4487	0.009	CG2017	8.07%	105.8000	7.4393
GS	CG2021	10.25%	123.8800	7.7755	0.000	CG2021	10.25%	124.4800	7.7758

However, if we look at the ZCYC for April 29, 2002, as given in the following picture, we find that for the maturity beyond 13.50 years, the yield showed a fall which is not in line with what has actually happened in the market. Hence, we find that the ZCYC is not really capturing the term structure of interest rate. And hence there will be less use of the less as long as steps are not taken to bring it in sync with the market. And any process that uses the ZCYC values as inputs is likely to produce incorrect results. Hence, we require some amount of stress testing for deriving a near accurate curve.



To make it more useful for the market participants, I suggest the following can be considered:

- a graph of the 1month forward rates as a function of start date (using an Act360 or Act365 basis): r(t) = [df(t) - df(t+30)]/{df(t+30) * 30/360). If there are bad smoothness properties of the curve, this will more likely expose it.

- one should check the larger errors in the theoretical & actual bond prices, and see if there is an explanation.
  They are usually something like
  a) issue is thinly traded and hasn't traded for a last few days
  b) an exceptionally large & liquid issue, so it trades richer than other issues
  c) an exceptionally illiquid issue, so it trades cheaper than other issues

Other explanations can generate reasons to trade: someone is getting out of a particular bond, or a squeeze, etc. If the forwards are good, and one is confident that the differences are understandable, then one can stand by the curve with confidence.

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